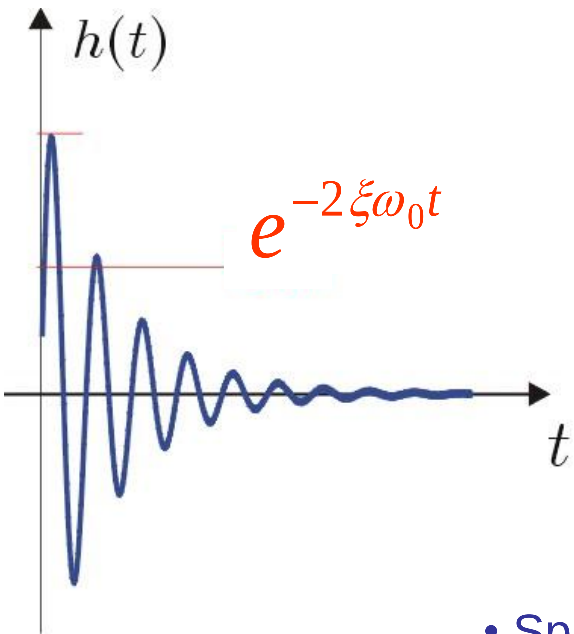
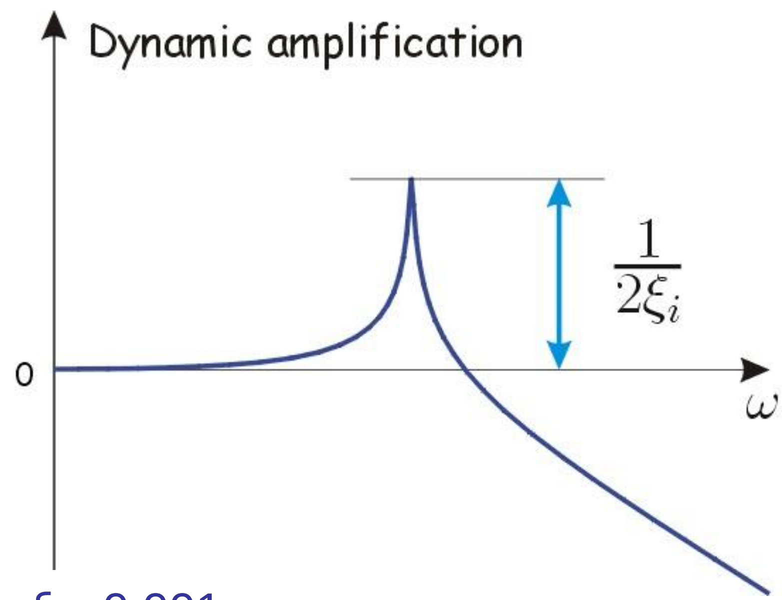
Lesson 4:

Vibration isolation

Vibration Damping
(= energy extraction)

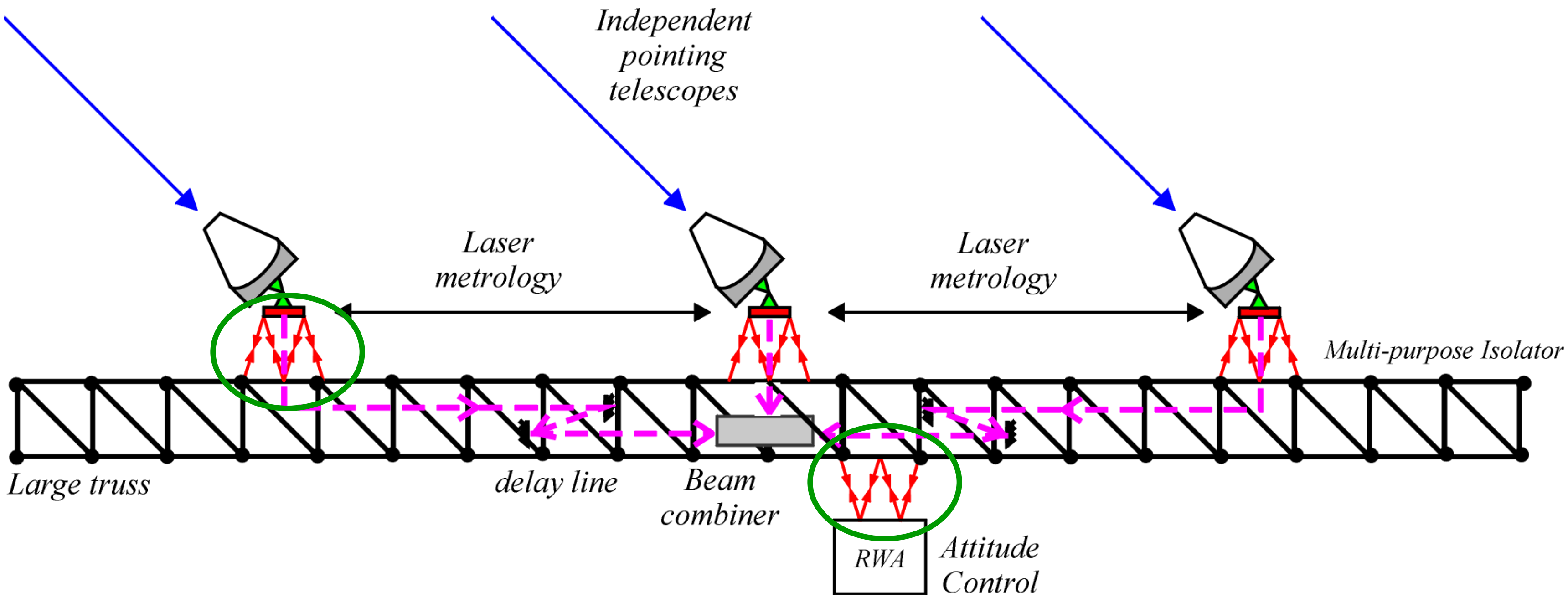
Damping ratio ξ_i:



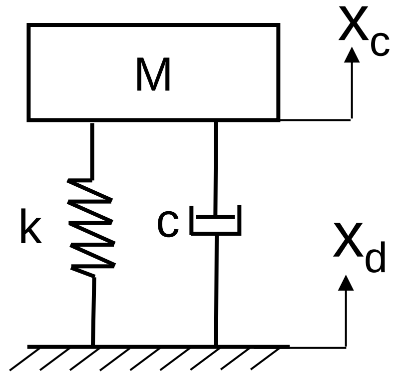


- Space structures: ξ_i~ 0.001
- Mechanical structures: ξi~ 0.01
- Civil engineering: ξi~ 0.1

Vibration Isolation



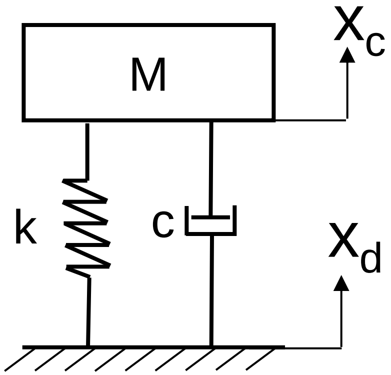
P.6.1 *Consider the passive isolator of Fig.6.1.b. Find the transmissibility $X_c(s)/X_d(s)$ of the isolation system.*



- Design for seismic vibration isolation.
- selection of c and k parameters.

Passive isolation

P.6.1 *Consider the passive isolator of Fig.6.1.b. Find the transmissibility* $X_c(s)/X_d(s)$ *of the isolation system.*



$$M\ddot{x}_c = \sum Forces$$

$$M\ddot{x}_c = c(\dot{x}_d - \dot{x}_c) + k(x_d - x_c)$$

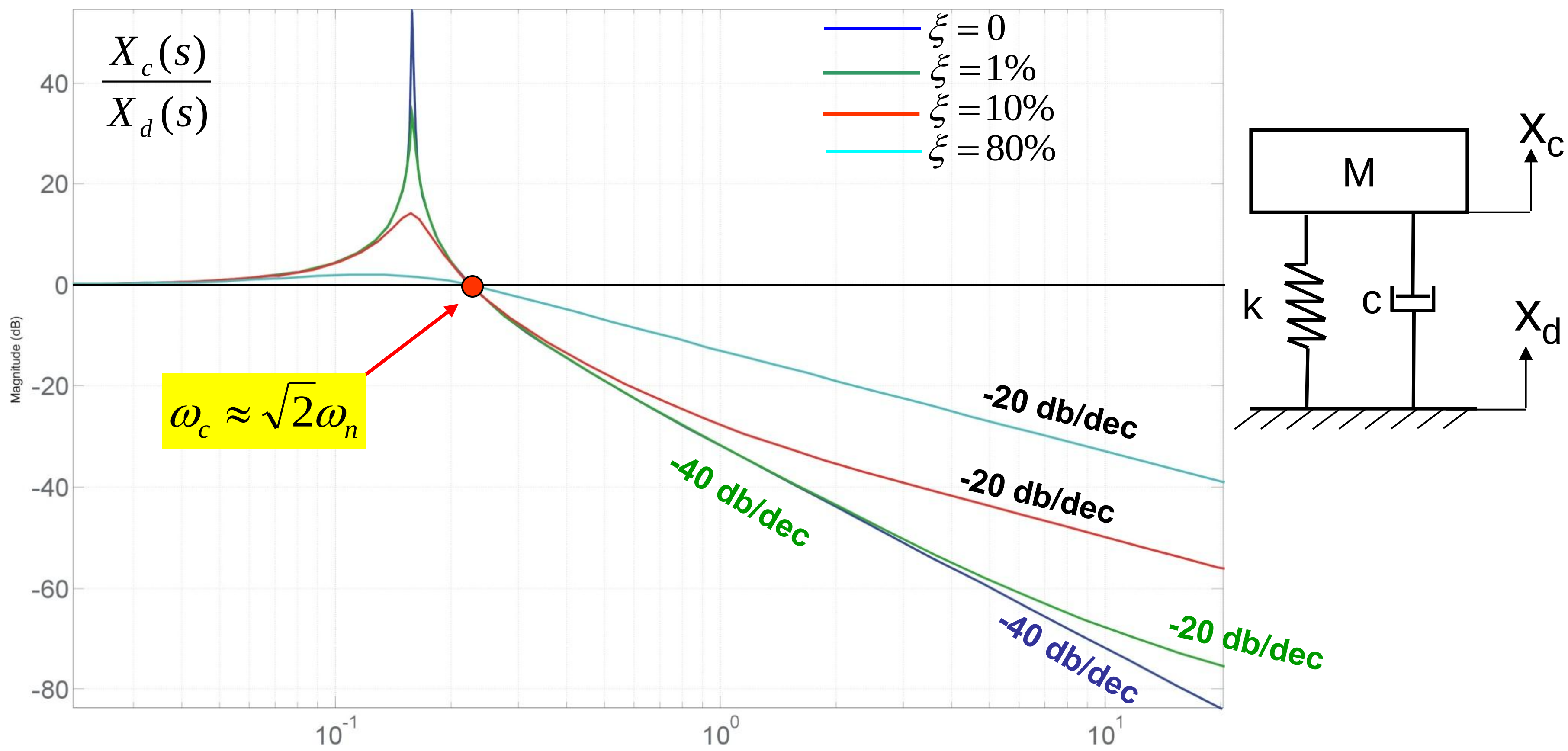
$$M\ddot{x}_c + c\dot{x}_c + kx_c = c\dot{x}_d + kx_d$$

$$G(s) = \frac{X_c(s)}{X_d(s)} = \frac{2\xi\omega_n s + \omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

One zero at $\frac{\omega_n}{2\xi}$

One pole at ω_n

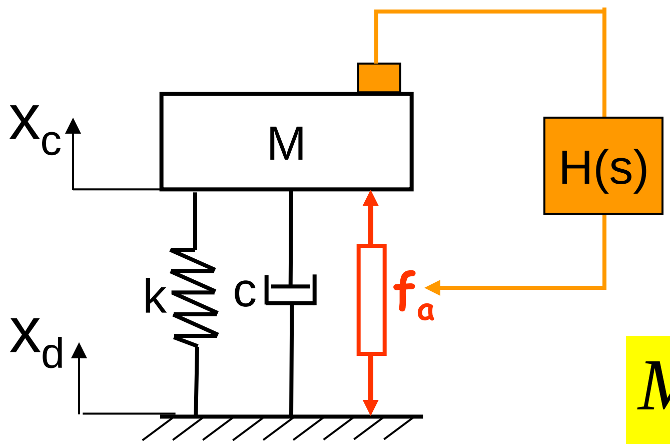
Passive isolation



$$G(s) = \frac{X_c(s)}{X_d(s)} = \frac{2\xi\omega_n s + \omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

Active Isolation : Sky-Hook damper

P.6.1 *Consider the passive isolator of Fig.6.1.b. Find the transmissibility $X_c(s)/X_d(s)$ of the isolation system.*



$$M\ddot{x}_c = \sum Forces$$

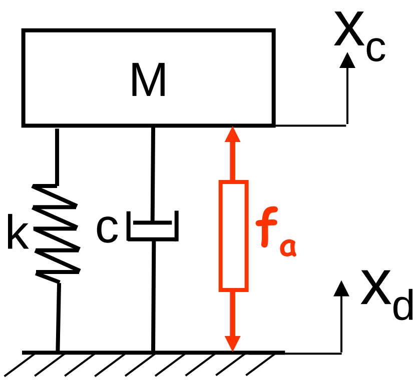
$$M\ddot{x}_c = c(\dot{x}_d - \dot{x}_c) + k(x_d - x_c) + f_a$$

$$M\ddot{x}_c + c\dot{x}_c + kx_c = c\dot{x}_d + kx_d + f_a$$

Choose f_a **proportionnel** to the velocity $\dot{x}_c$

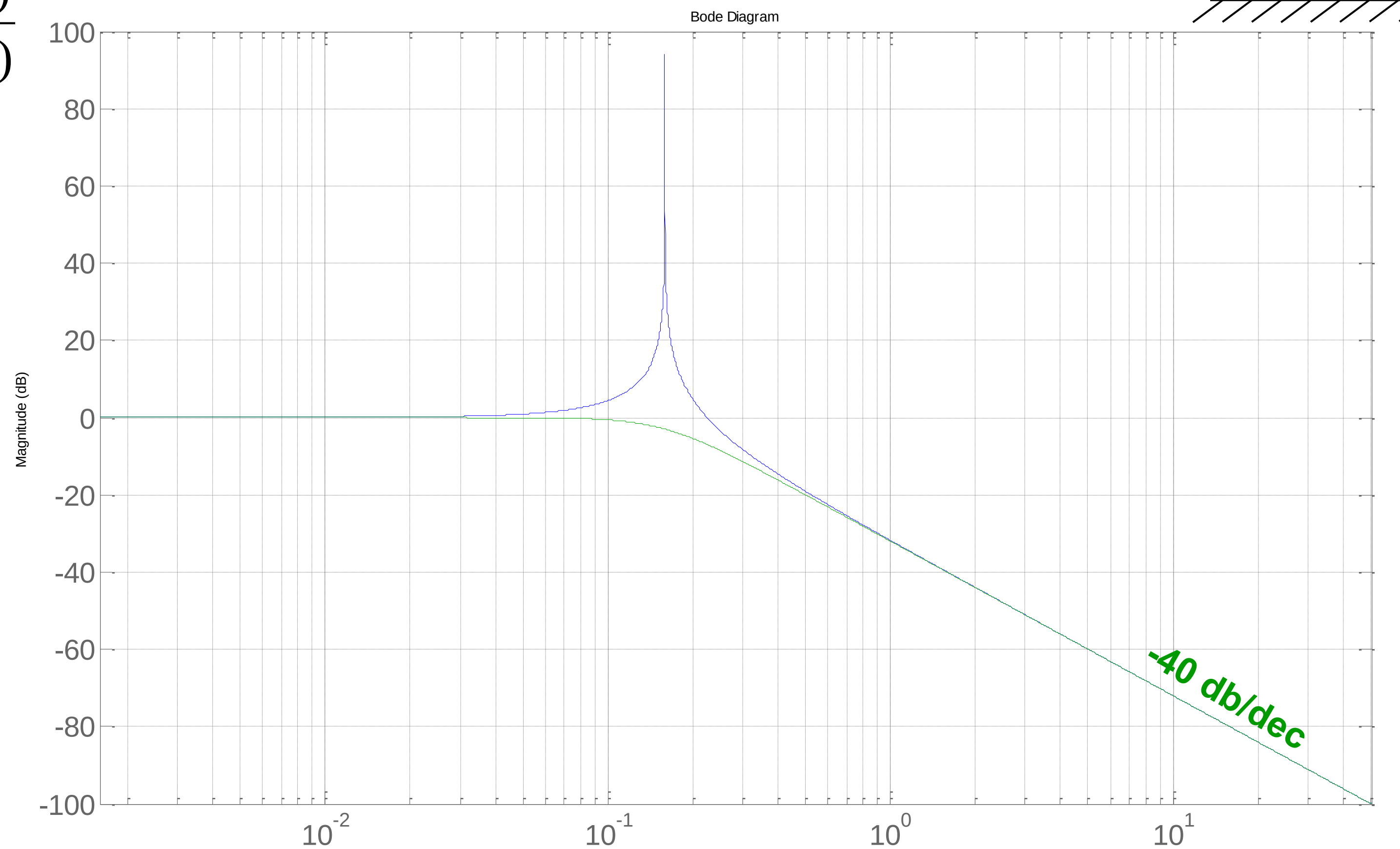
$$f_a \propto g\dot{x}_c \quad \Rightarrow \quad G(s) = \frac{X_c(s)}{X_d(s)} = \frac{cs + k}{Ms^2 + (c+g)s + k}$$

Active Isolation : Sky-Hook damper

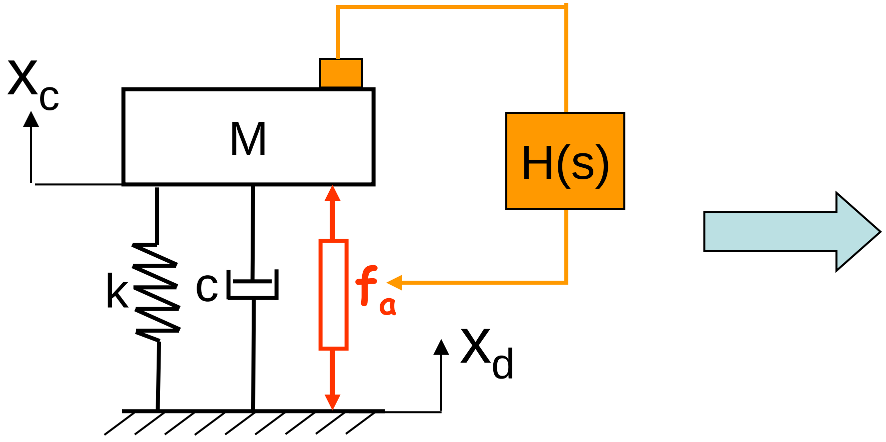


Accelerometer sensor $\longrightarrow H(s) = -g\dfrac{1}{s}\ddot{X}_c \equiv -gsX_c$

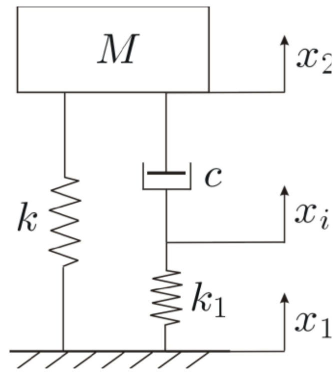
$$\frac{X_c(s)}{X_d(s)}$$



Active Isolation : Sky-Hook damper



Active



Passive

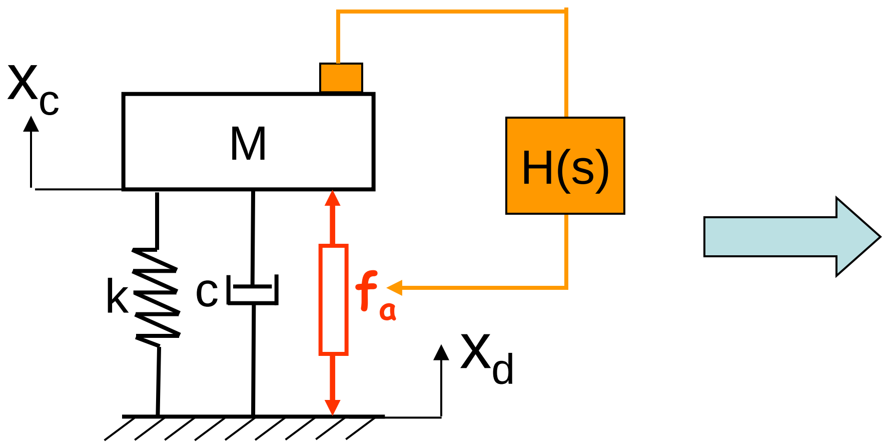
$$\begin{bmatrix} Ms^2+cs+k & -cs \\ -cs & k_1+cs \end{bmatrix} \begin{Bmatrix} x \\ x_1 \end{Bmatrix} = \begin{Bmatrix} k \\ k_1 \end{Bmatrix} x_0$$

Transmissibility:

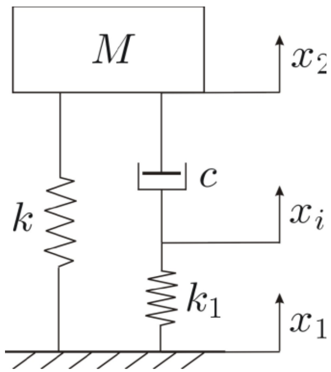
$$\omega_n^2 = \frac{k}{M}, \qquad 2\,\xi\omega_n = \frac{c}{M}, \qquad \tau_1 = \frac{c}{k_1}, \qquad \kappa = \frac{k_1}{k}$$

$$\frac{x}{x_0} = \frac{1+\tau_1 s + 2\xi s/\omega_n}{\tau_1 s^3/\omega_n^2 + s^2/\omega_n^2 + \tau_1 s + 2\xi s/\omega_n + 1}$$

Active Isolation : Sky-Hook damper



Active



Passive

$$(k_1 + cs)(Ms^2 + k) + k_1 cs = 0$$

$$c(\dot{x}_2 - \dot{x}_i) = k(x_i - x_2),$$

$$G(s) = \frac{X_2}{X_1} = \frac{(k_1 + cs)k + k_1 cs}{(k_1 + cs)(Ms^2 + k) + k_1 cs} \tag{11}$$

Active Isolation : Sky-Hook damper

$$G(s) = \frac{X_2}{X_1} = \frac{(k_1 + cs)k + k_1 cs}{(k_1 + cs)(Ms^2 + k) + k_1 cs}$$

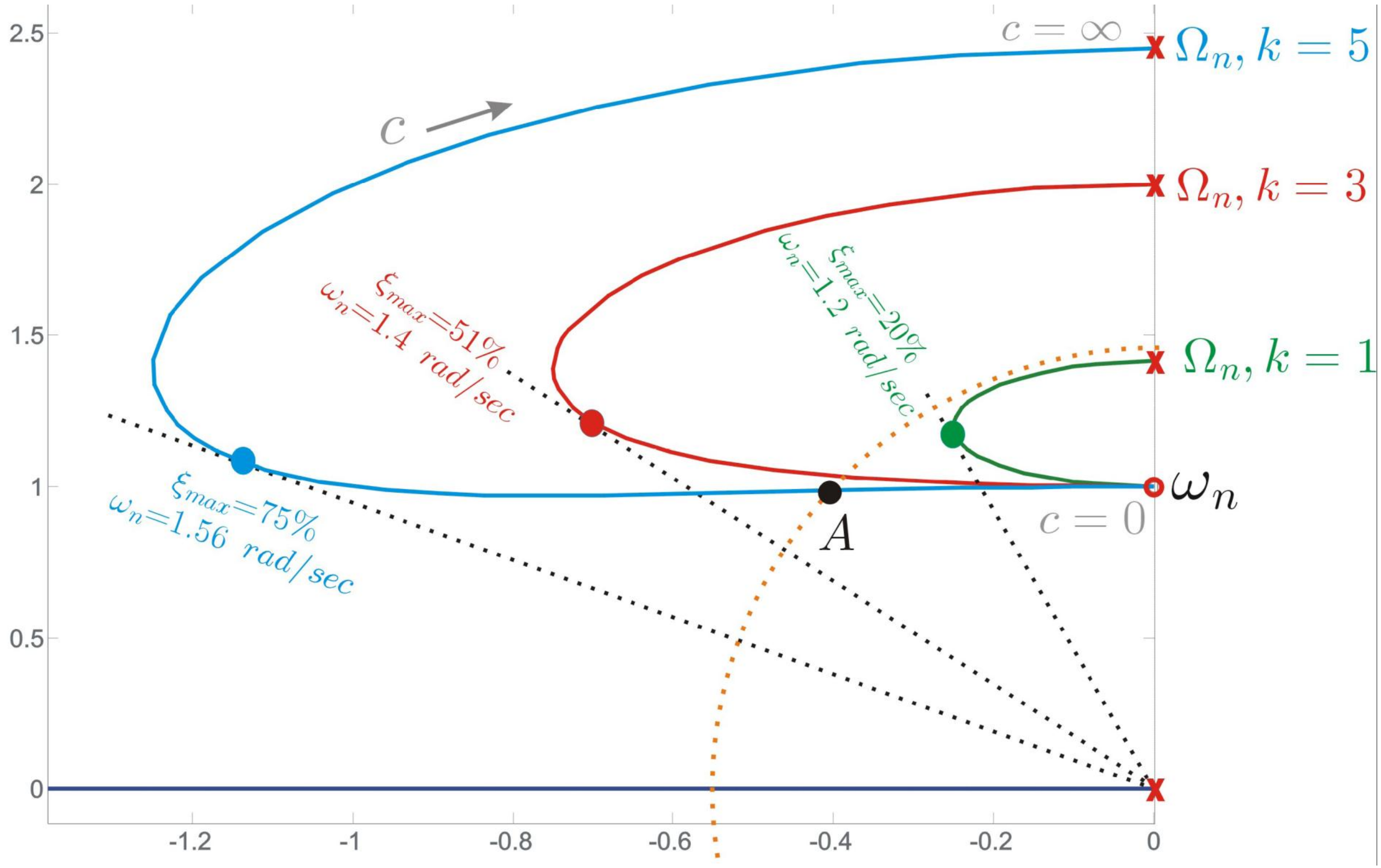
Characteristic polinomial

$$(k_1 + cs)(Ms^2 + k) + k_1 cs = 0$$

$$1 + g\frac{\prod(s - z_i)}{\prod(s - p_i)} = 1 + \frac{k_1}{c}\frac{(s^2 + \omega_n^2)}{s(s^2 + \Omega_n^2)} = 0, \quad \omega_n = \sqrt{\frac{k}{M}}, \Omega_n = \sqrt{\frac{k + k_1}{M}}$$

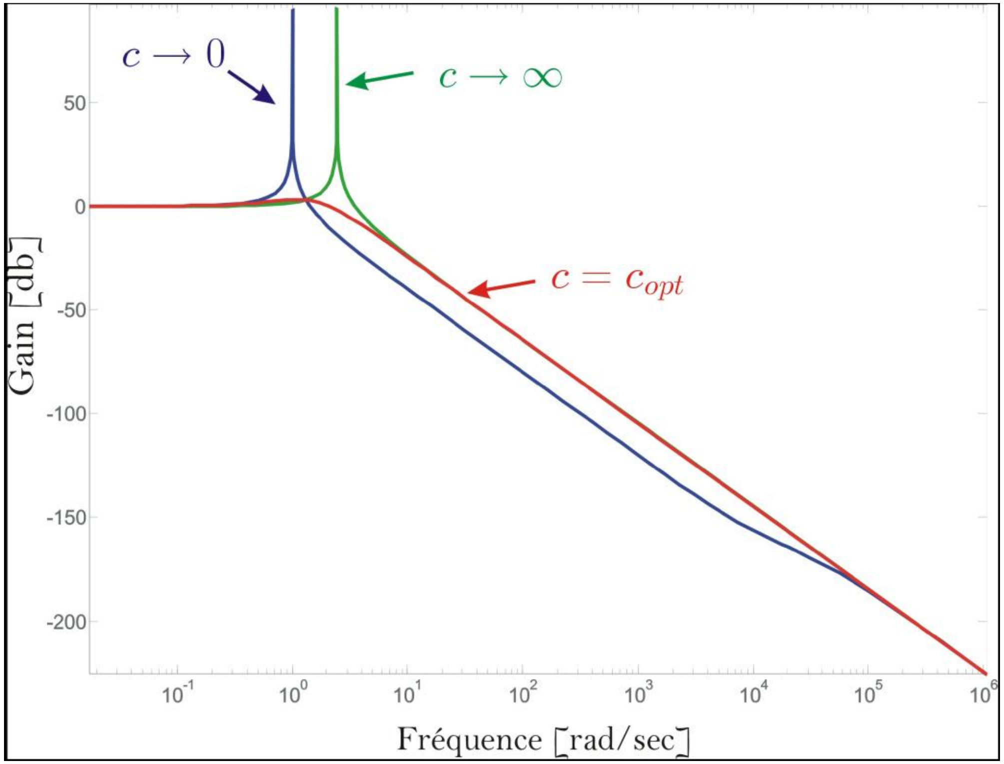
Use the same principle as the root locus

$$g = \frac{k_1}{c} = 0 \rightarrow \infty \quad \begin{cases} c : \infty \rightarrow 0 \\ \text{tune} \quad k_1 \end{cases} \qquad \omega_n = \sqrt{\frac{k}{M}}, \Omega_n = \sqrt{\frac{k + k_1}{M}}$$



$c = \infty$
$\Omega_n, k = 5$
c
$\Omega_n, k = 3$
ξ_{max}=51%
ω_n=1.4 rad/sec
ξ_{max}=20%
ω_n=1.2 rad/sec
$\Omega_n, k = 1$
ω_n
ξ_{max}=75%
ω_n=1.56 rad/sec
A
$c = 0$
2.5
2
1.5
1
0.5
0
-1.2
-1
-0.8
-0.6
-0.4
-0.2
0

Active Isolation : Sky-Hook damper



For a given value of k_1

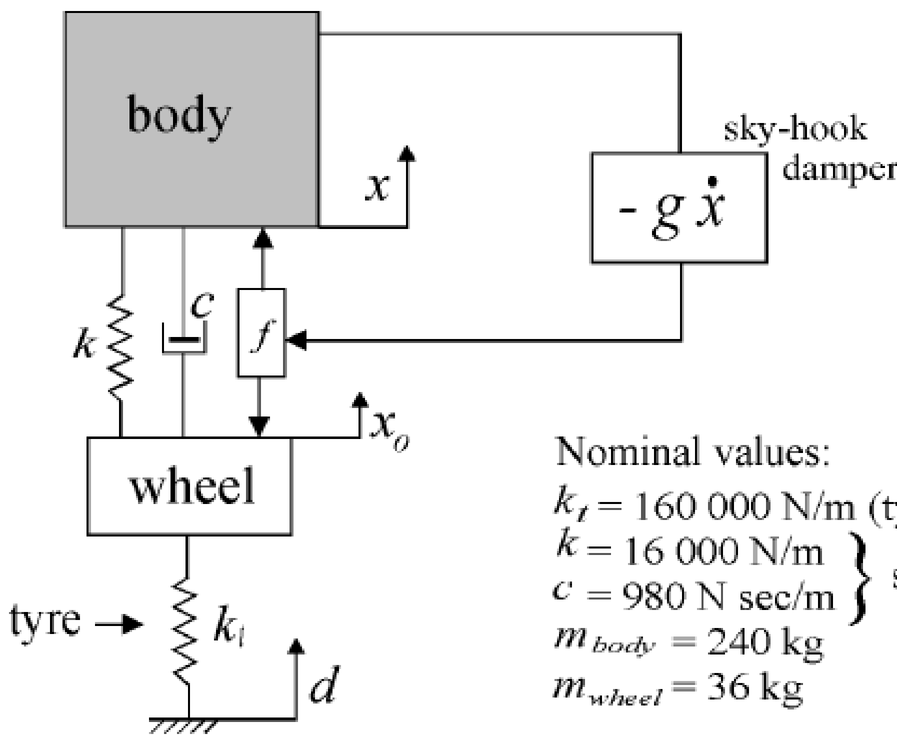
$$c_{opt} = k_1 \frac{\omega_n^{1/2}}{\Omega_n^{3/2}}$$

Increasing k :

- increse the damping: **nice**
- increase the bandwidth: **bad !**
- **We can consider the point A as an optimum**

} Trade-off

P.6.8 (1) Develop a state space model of the vehicle suspension of Fig.6.18 (use x, $\dot{x}$, x_0 and $\dot{x}_0$ as variables).
(2) Plot the amplitude of the FRF $|\ddot{x}/\dot{d}|$ of the passive suspension for various values of the damping coefficient c. Compare with Fig.6.19.
(3) Consider the active suspension with a sky-hook damper control law. Plot the amplitude of the FRF $|\ddot{x}/\dot{d}|$ for the nominal parameters and various values of the gain g. Compare the frequency content of the absolute velocity of the body $\dot{x}$ with that of the relative velocity $(\dot{x} - \dot{x}_0)$.



Nominal values:
k_t = 160 000 N/m (tyre)
k = 16 000 N/m
c = 980 N sec/m } suspension
m_{body} = 240 kg
m_{wheel} = 36 kg

1 : State space model

$$M\ddot{X} + C\dot{X} + KX = b_{in} f$$

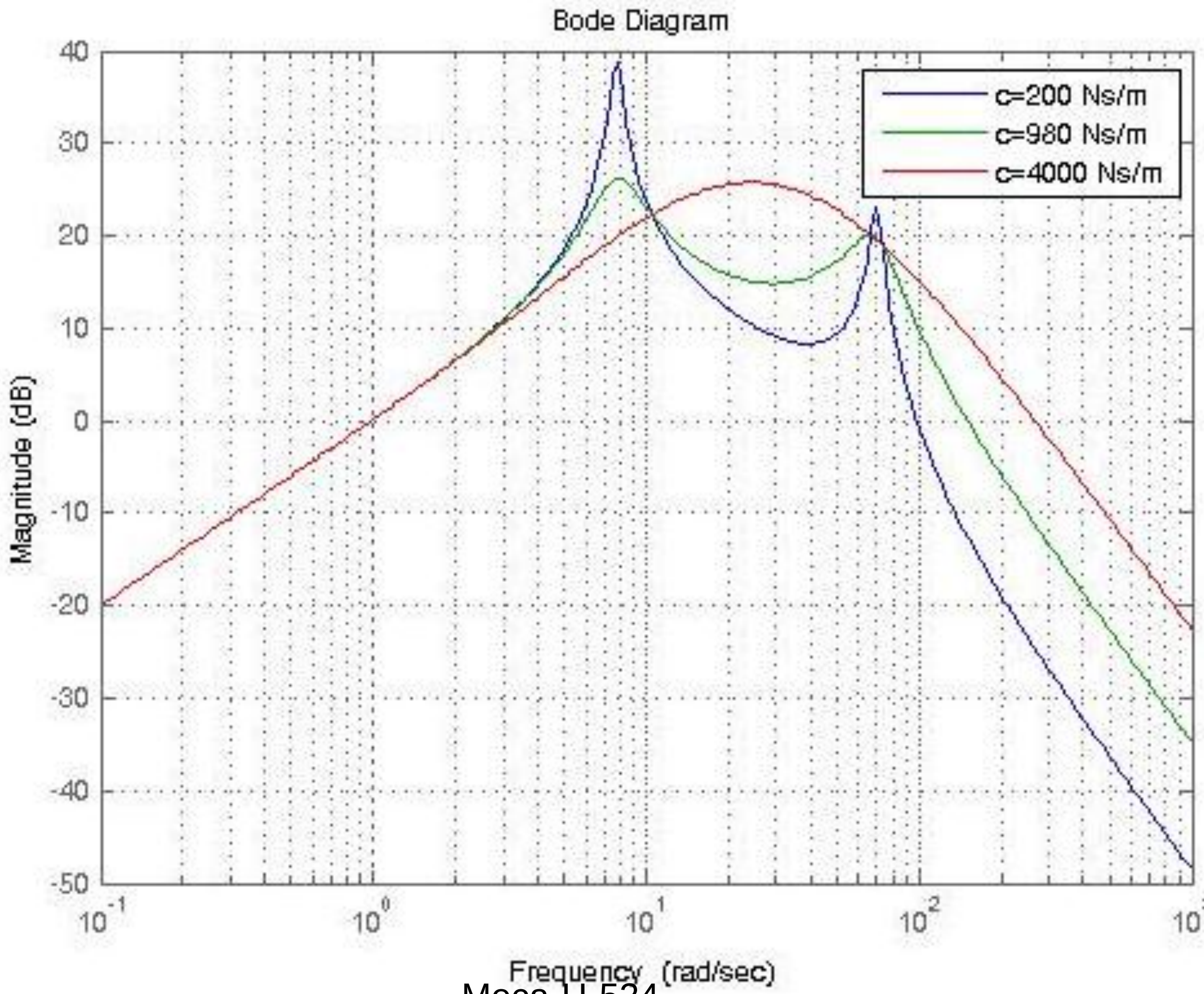
$$\Rightarrow \begin{Bmatrix} \dot{X} \\ \ddot{X} \end{Bmatrix} = \begin{bmatrix} 0 & I_n \\ -M^{-1}K & -M^{-1}C \end{bmatrix} \begin{Bmatrix} X \\ \dot{X} \end{Bmatrix} + \begin{bmatrix} 0 \\ M^{-1}b_{in} \end{bmatrix} f$$

$$y = b_1^T X + b_2^T \dot{X} + b_3^T f$$

$$\Rightarrow \quad y = \begin{bmatrix} b_1^T & b_2^T \end{bmatrix} \begin{Bmatrix} X \\ \dot{X} \end{Bmatrix} + b_3^T f$$

2 : Frequency response

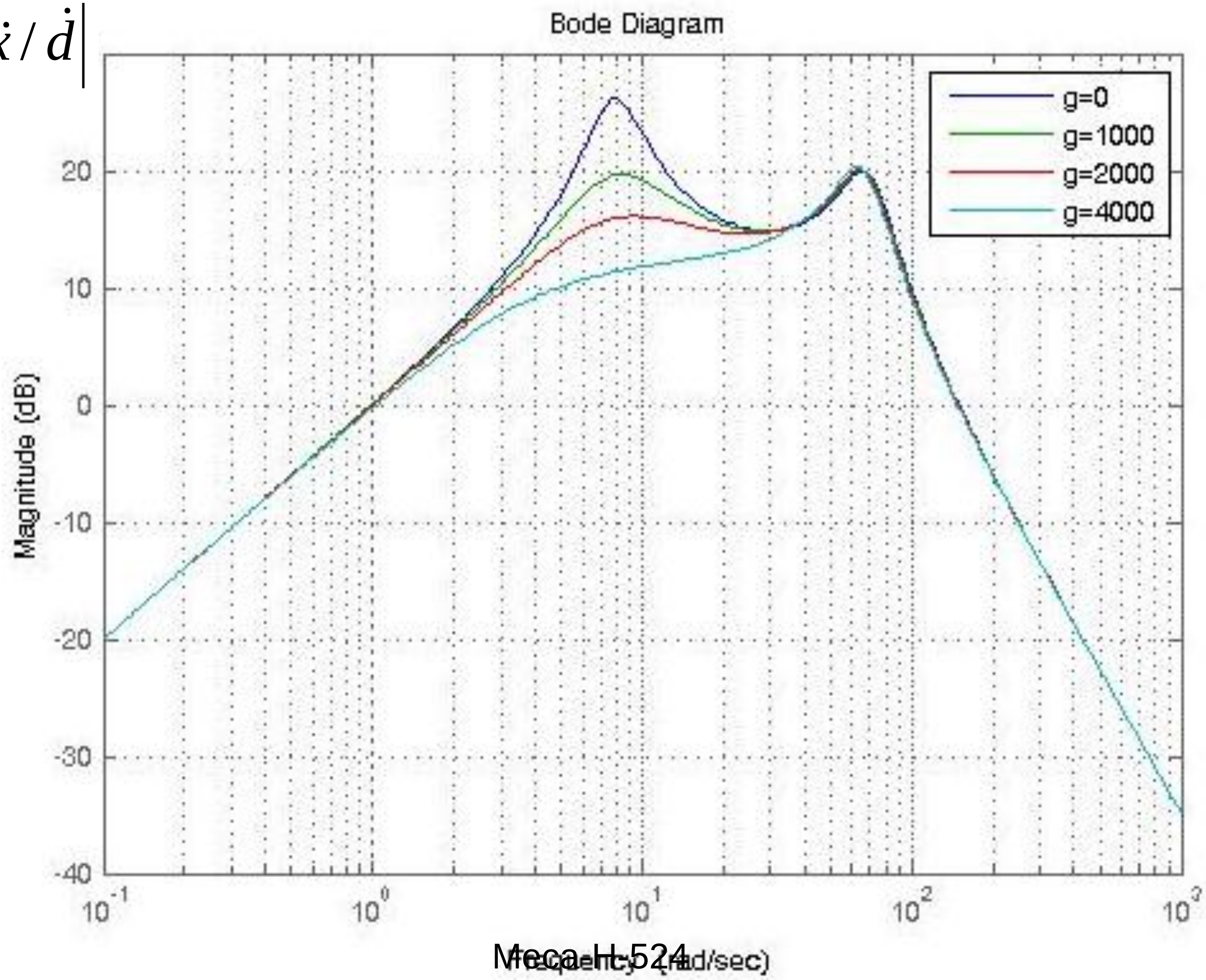
$$\left|\ddot{x} / \dot{d}\right|$$



3 : sky-hook isolator

Use the *feedback()* function from Matlab

$$\left|\ddot{x}/\dot{d}\right|$$



3 : sky-hook isolator