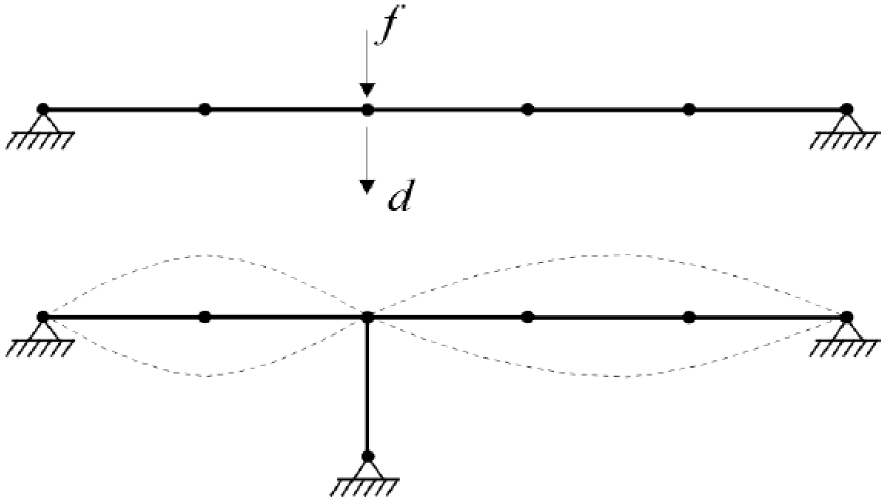
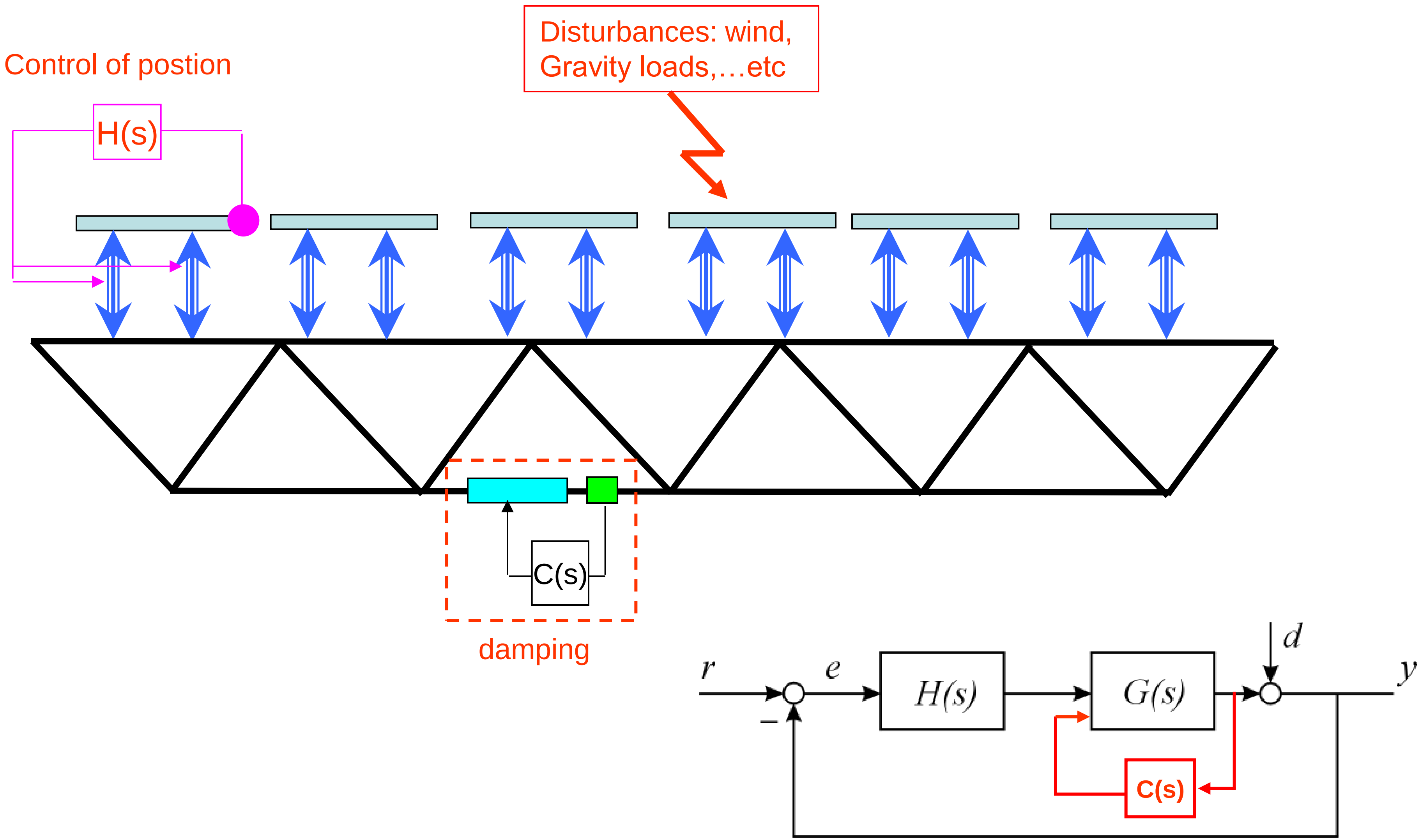
Lesson 3

Active damping, Colocolated control and modeling

- Active damping is different from disturbance rejection
- We try to control the whole structure with a finite number of actuators and sensors
- The control system must be designed to dissipate the maximum of energy



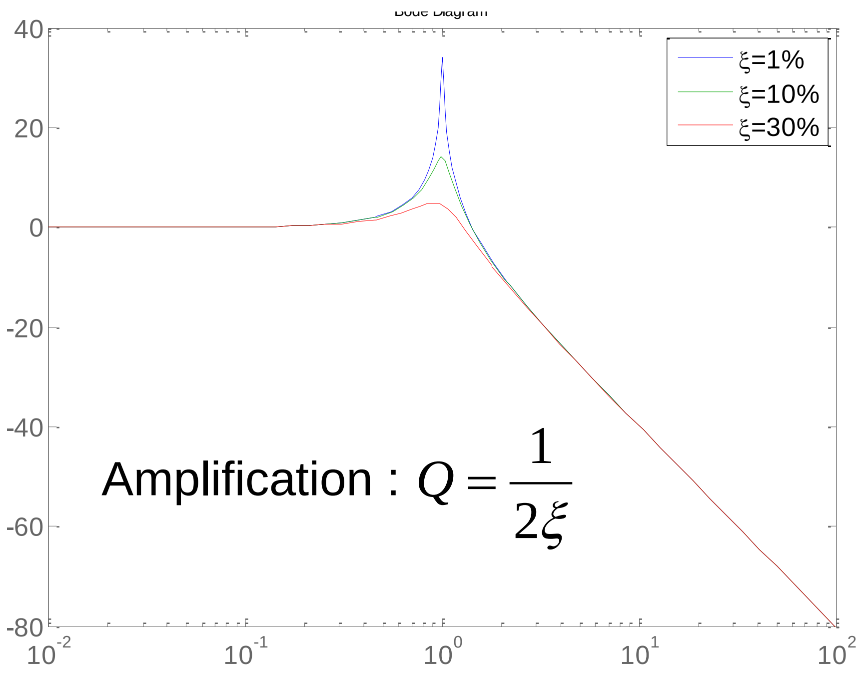
Exemple of active damping



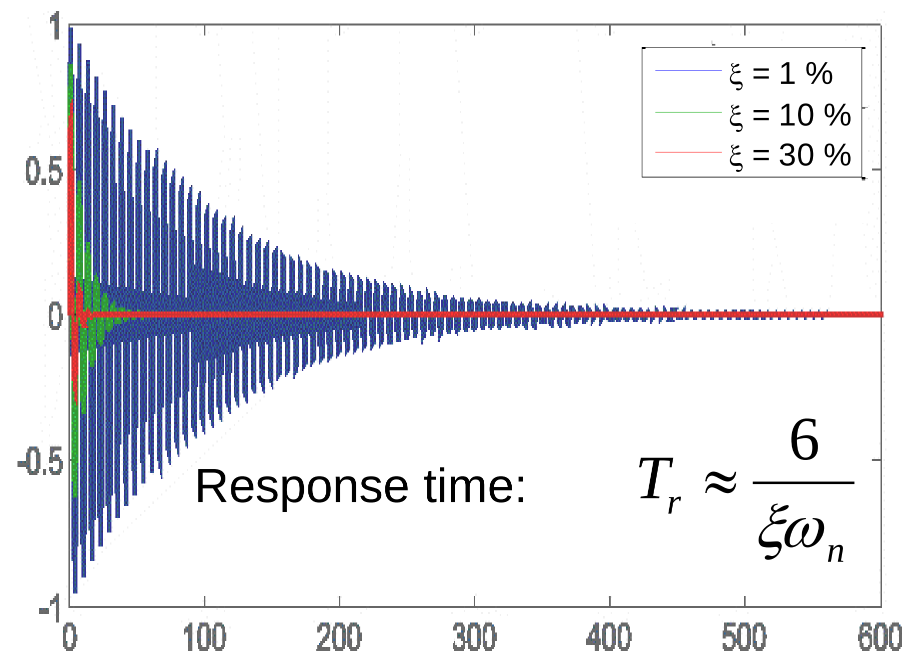
Effect of damping

$$G(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

Impulse response:



Amplification : $Q = \frac{1}{2\xi}$



Response time: $T_r \approx \frac{6}{\xi\omega_n}$

P.5.2 Suppose that a mechanical structure is equipped with a point-force actuator and a collocated displacement sensor. Discuss the following implementations of the Lead and Direct Velocity Feedback compensators :

$$D(s) = s$$
$$D(s) = 1 + Ts$$
$$D(s) = \frac{s}{s+a}$$
$$D(s) = \frac{Ts+1}{\alpha Ts+1} \quad (\alpha < 1)$$
$$D(s) = \frac{\omega_f^2 s}{s^2 + 2\xi_f \omega_f s + \omega_f^2}$$

Discuss the conditions under which these compensators would be applicable for active damping.

Principal :

The frequency response of the controler (amplitude + phase) informs us about:

1. Its efficiency for damping
2. Implementing possibilities

A. Damping:

The control force must to be proportional to the velocity to dissipate energy, i.e.:

$$u(t) = gv(t) = g\frac{d}{dt}y(t)$$

→The conroler must behave like a pure derivative near the frequency band of interest. In Laplace variable, this gives :

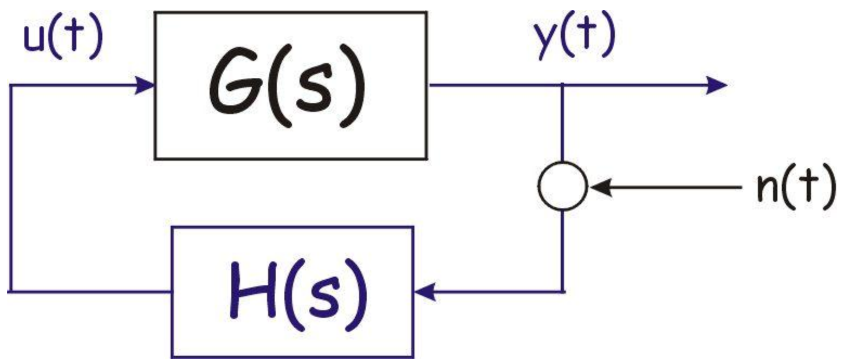
$$U(s) = g\ s\ Y(s)$$

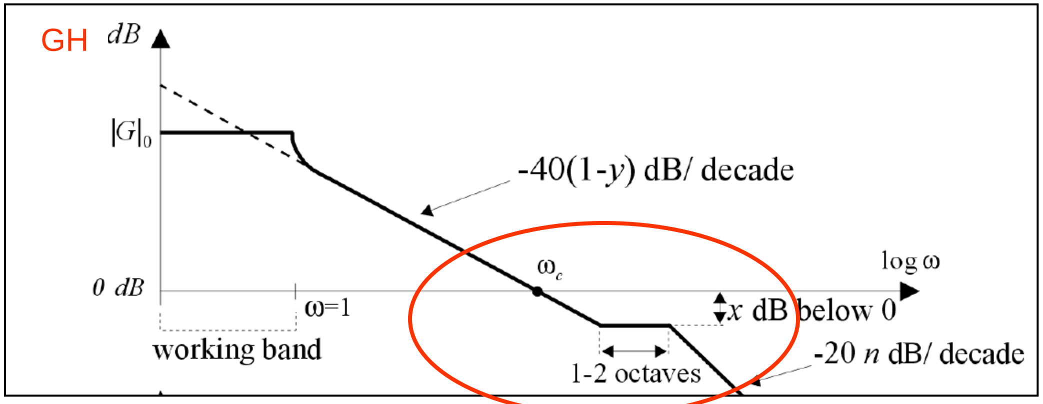
→ The ideal controler has a phase of 90° and a slope of +20dB/décade.

B. Implementation :

At high frequncies, we must have :

1. Noise : H(s) not too big (for *n(t)).*
2. Stability : G(s) * H(s) small enough.





Modeling of G(s)

P.2.4 *Consider a simply supported beam with the following properties:* $l = 1m$, $m = 1kg/m$, $EI = 10.266\ 10^{-3} Nm^2$. *It is excited by a point force at* $x_a = l/4$.

(a) Assuming that a displacement sensor is located at $x_s = l/4$ *(collocated) and that the system is undamped, plot the transfer function for a increasing number of modes, with and without quasi-static correction for the high-frequency modes. Comment on the variation of the zeros with the number of modes and on the absence of mode 4.*

Note: To evaluate the quasi-static contribution of the high-frequency modes, it is useful to recall that the static displacement at $x = \xi$ *created by a unit force applied at* $x = a$ *on a simply supported beam is*

$$\delta(\xi, a) = \frac{(l-a)\xi}{6lEI}[a(2l-a) - \xi^2] \quad (\xi \le a)$$

$$\delta(\xi, a) = \frac{a(l-\xi)}{6lEI}[\xi(2l-\xi) - a^2] \quad (\xi > a)$$

The symmetric operator $\delta(\xi, a)$ *is often called "flexibility kernel" or Green's function.*

(b) Including three modes and the quasi-static correction, draw the Nyquist and Bode plots and locate the poles and zeros in the complex plane for a uniform modal damping of $\xi_i = 0.01$ *and* $\xi_i = 0.03$.

(c) Do the same as (b) when the sensor location is $x_s = 3l/4$. *Notice that the interlacing property of the poles and zeros no longer holds.*

How to compute G(*jω*) for a mechanical structure?

1. The direct way is to compute G(s) point by point :

$$\left(Ms^2 + Cs + K\right)X = b_{in}F$$

$$\Rightarrow \quad x_{out} = b_{out}^T X$$

$$= b_{out}^T \left(Ms^2 + Cs + K\right)^{-1} b_{in} F$$

- The matrices M,K,C (of dimensions n*n) are obtained in different ways (finite element modal, écriture directe dans le cas de petits modèles,…)
- The vectors b = [0 0 .. 1 .. 0 0]^T define the topology (location) of the actuators and the sensors.

BUT: this technique is very slow and inefficient for big models.

How to compute G($j\omega$) for a mechanical structure?

2. Usualyn we prefer to use a modal decomposition:

$$G(s) = \sum_{i=1}^{n} \frac{b_{out}^T \phi_i \phi_i^T b_{in}}{\mu_i \left(s^2 + 2\xi_i \omega_i s + \omega_i^2\right)}$$

In the case of supported beam:

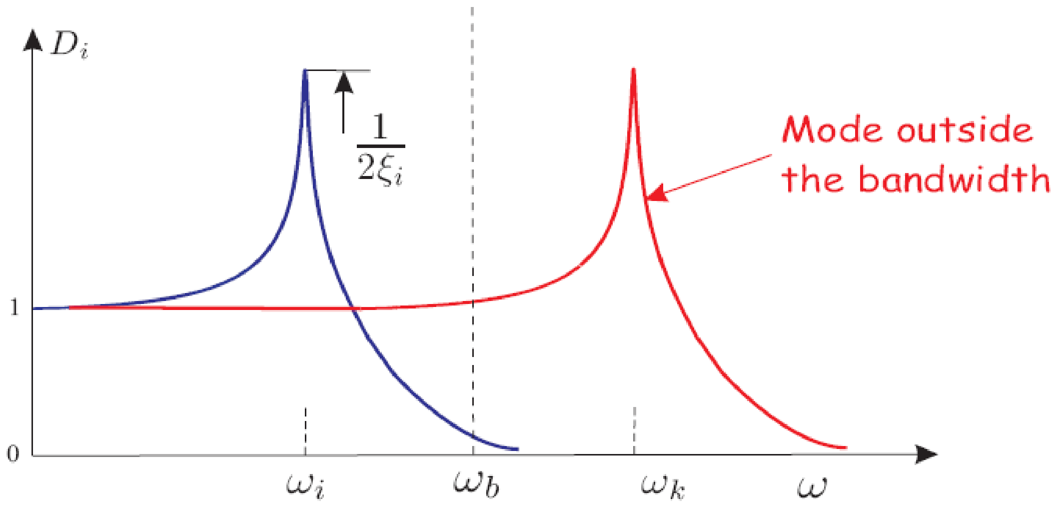
$$\omega_n^2 = (n\pi)^4 \frac{EI}{ml^4} \qquad (2.43)$$

$$\phi_n(x) = \sin \frac{n\pi x}{l} \qquad (2.44)$$

$$\mu_n = ml/2$$

How to compute G(*jω*) for a mechanical structure?

3. What if we want/can to compute only *m* modes ?

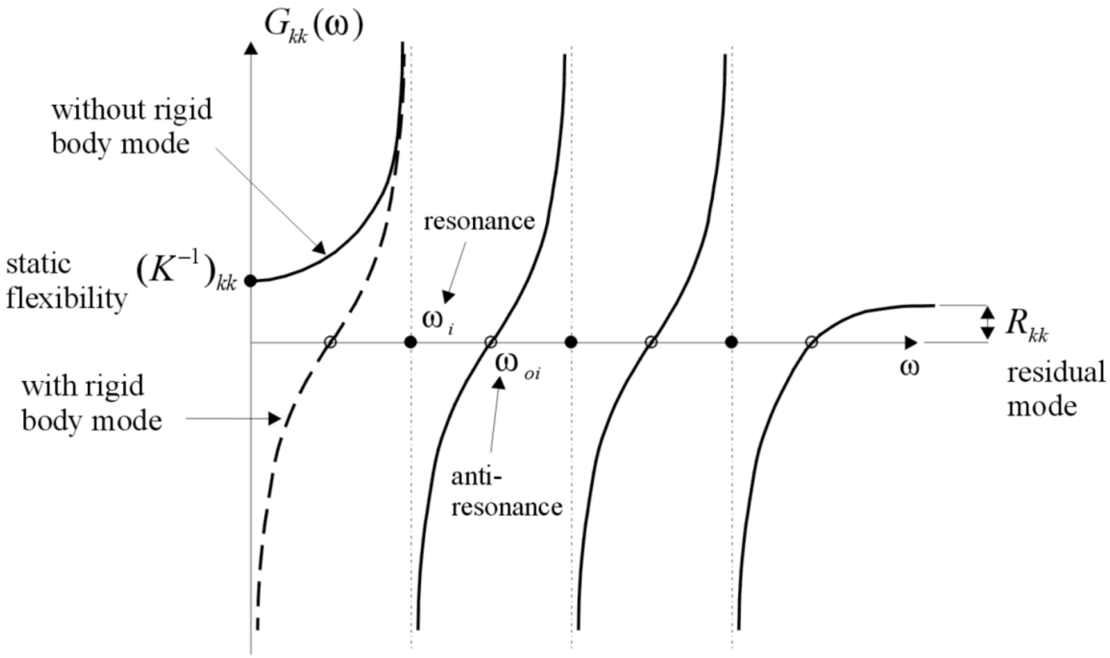


We neglect the « dynamic » part of the following modes :

$$\frac{\phi_i \phi_i^T}{\mu_i \left(s^2 + 2\xi_i \omega_i s + \omega_i^2\right)} \approx \frac{\phi_i \phi_i^T}{\mu_i \omega_i^2} \quad \Rightarrow \quad G(s) \approx \sum_{i=1}^{m} \frac{b_{out}^T \phi_i \phi_i^T b_{in}}{\mu_i \left(s^2 + 2\xi_i \omega_i s + \omega_i^2\right)} + C$$

How to compute G(*jω*) for a mechanical structure?

4. What is the impact of the quasi-static correction?

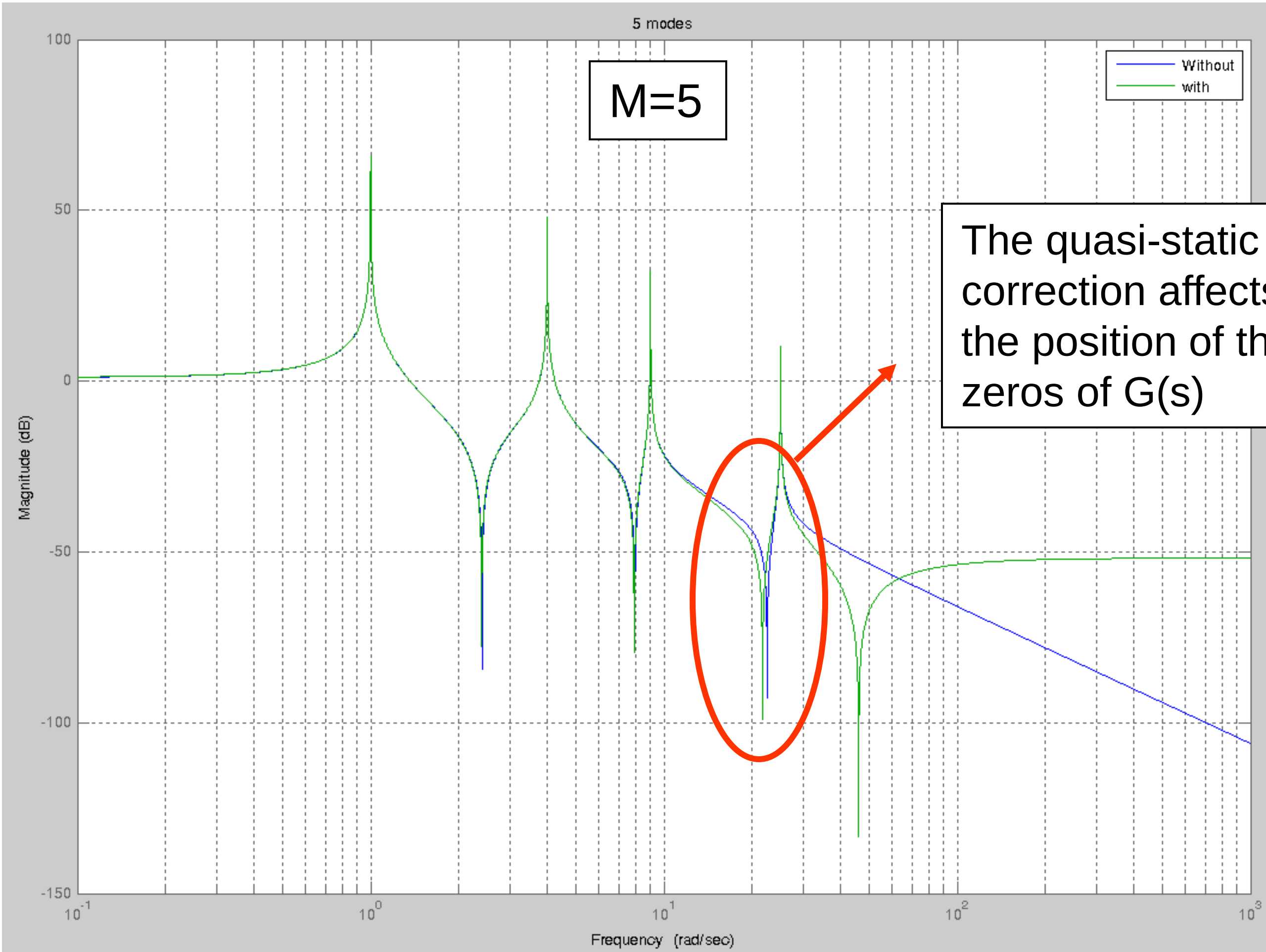


The high frequency modes doesn't affect the position of the poles, **But they have a big effect on the position of the zeros.**

Ex. 2.4

Static response

$$G(\omega) \simeq \sum_{i=1}^{m} \frac{\phi_i \phi_i^T}{\mu_i(\omega_i^2 - \omega^2 + 2j\xi_i\omega_i\omega)} + K^{-1} - \sum_{i=1}^{m} \frac{\phi_i \phi_i^T}{\mu_i \omega_i^2} \qquad (2.22)$$

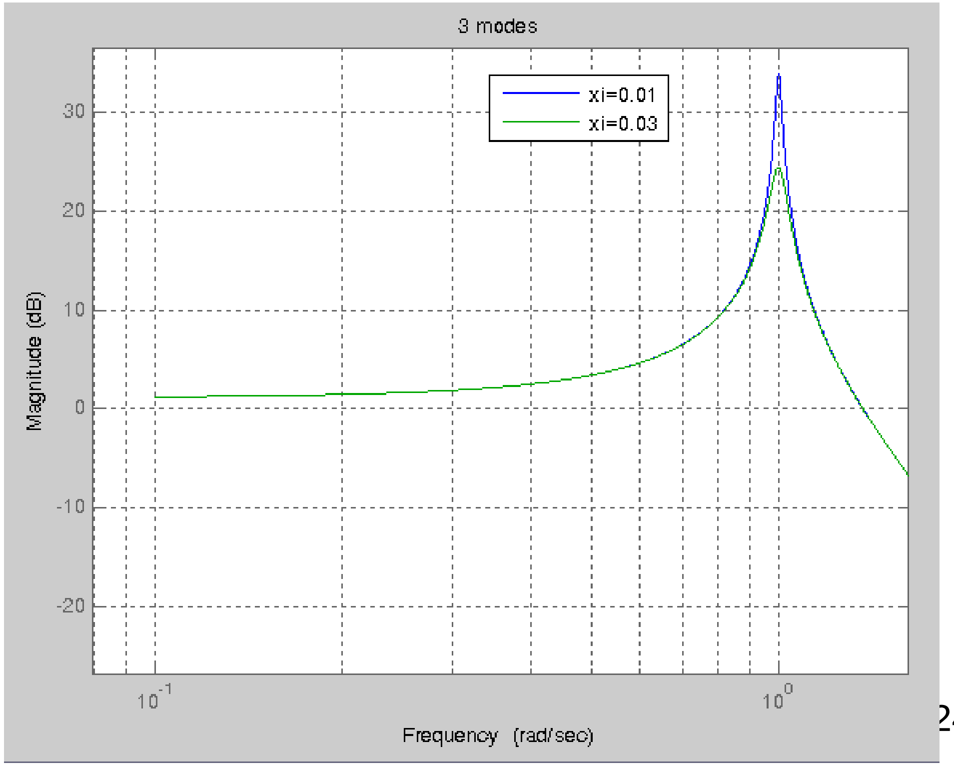


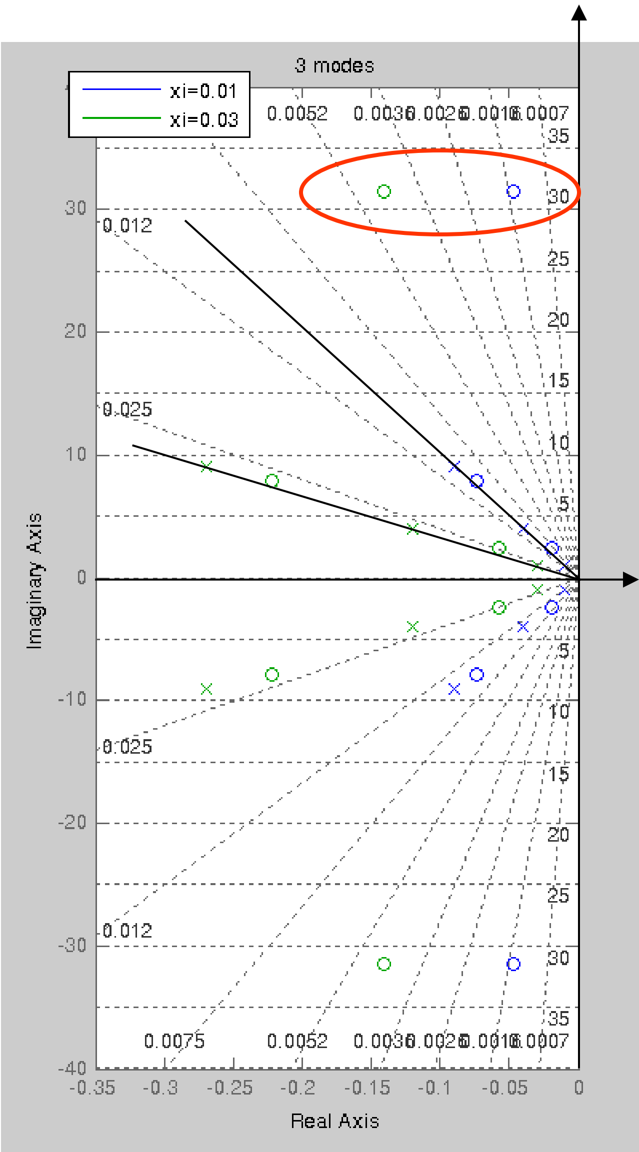
The quasi-static correction affects only the position of the zeros of G(s)

P.2.4 *Consider a simply supported beam with the following properties:* $l = 1m$, $m = 1kg/m$, $EI = 10.266\ 10^{-3} Nm^2$. *It is excited by a point force at* $x_a = l/4$.

(b) Including three modes and the quasi-static correction, draw the Nyquist and Bode plots and locate the poles and zeros in the complex plane for a uniform modal damping of $\xi_i = 0.01$ *and* $\xi_i = 0.03$.

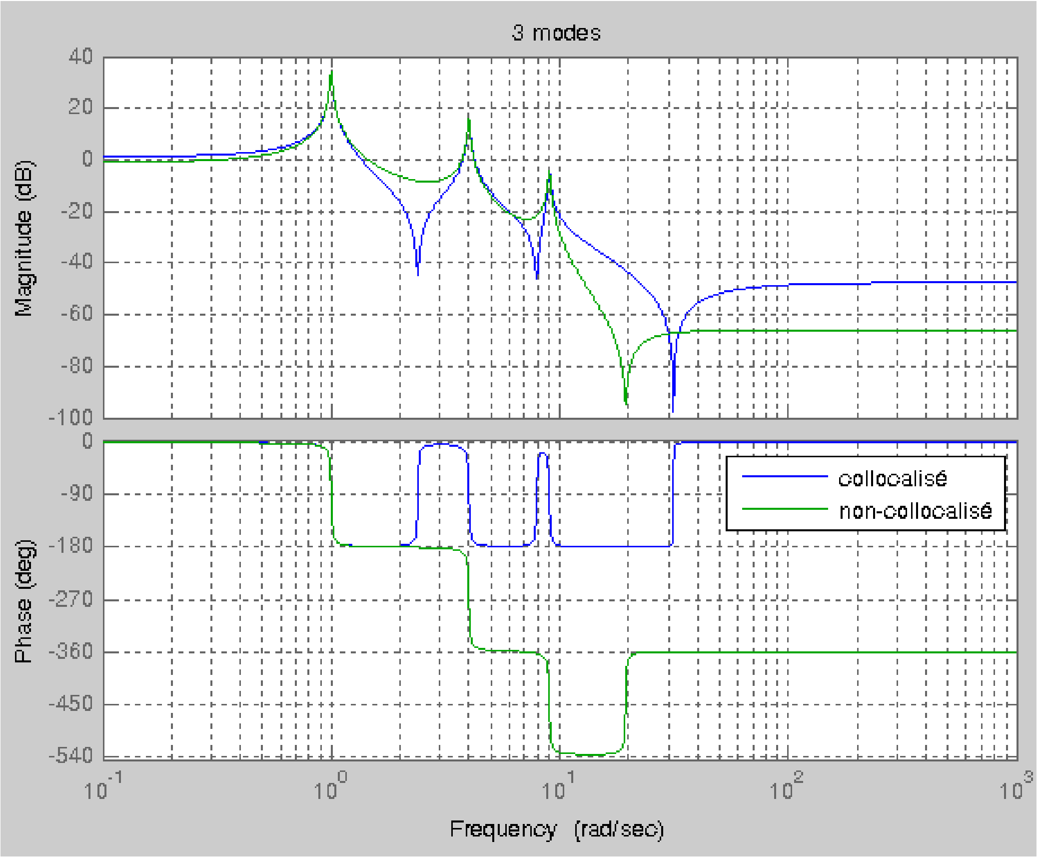
(c) Do the same as (b) when the sensor location is $x_s = 3l/4$. *Notice that the interlacing property of the poles and zeros no longer holds.*





P.2.4 (c) : When the sensor and the actuator are not in the sam location

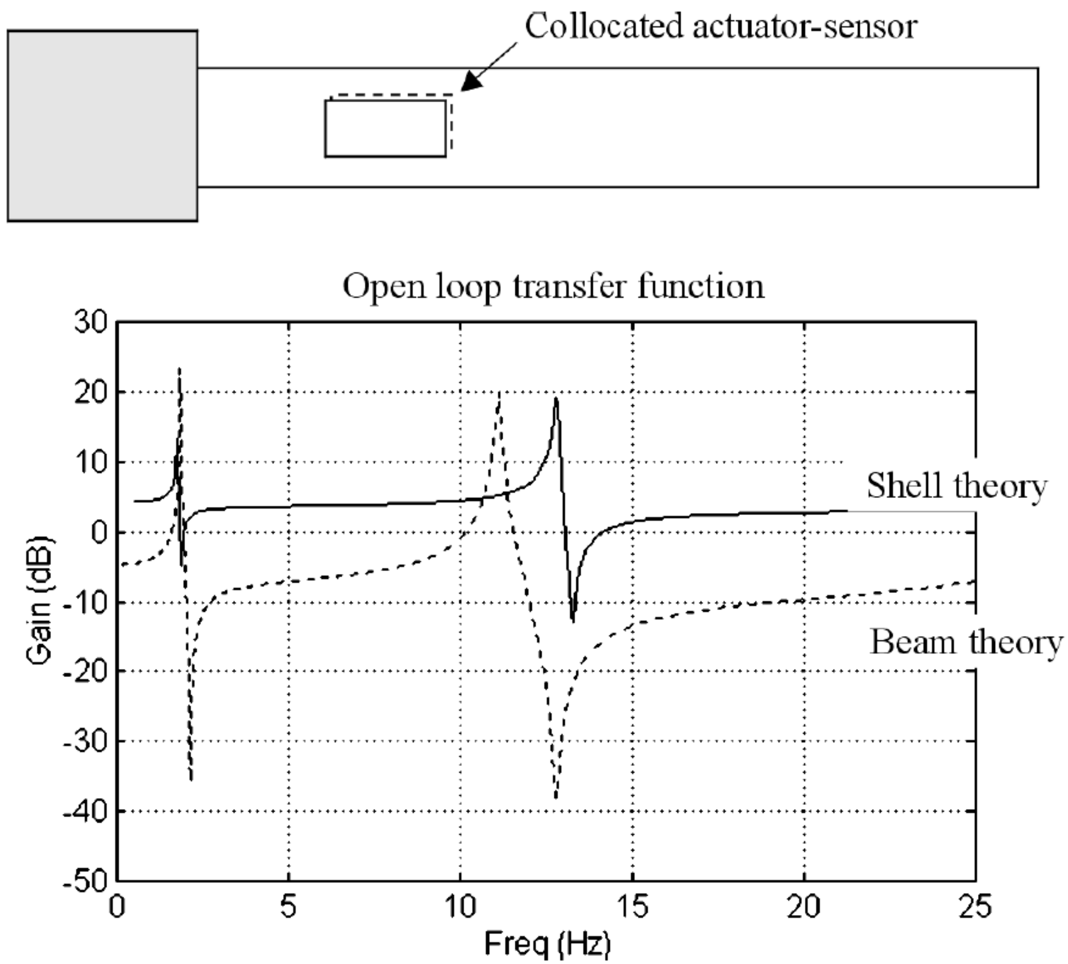
- The poles doesn't change (inherent property of he system, independant from the sensor)
- The zeros are very differents:
  - No alternance pole/zero => Problem with the phase



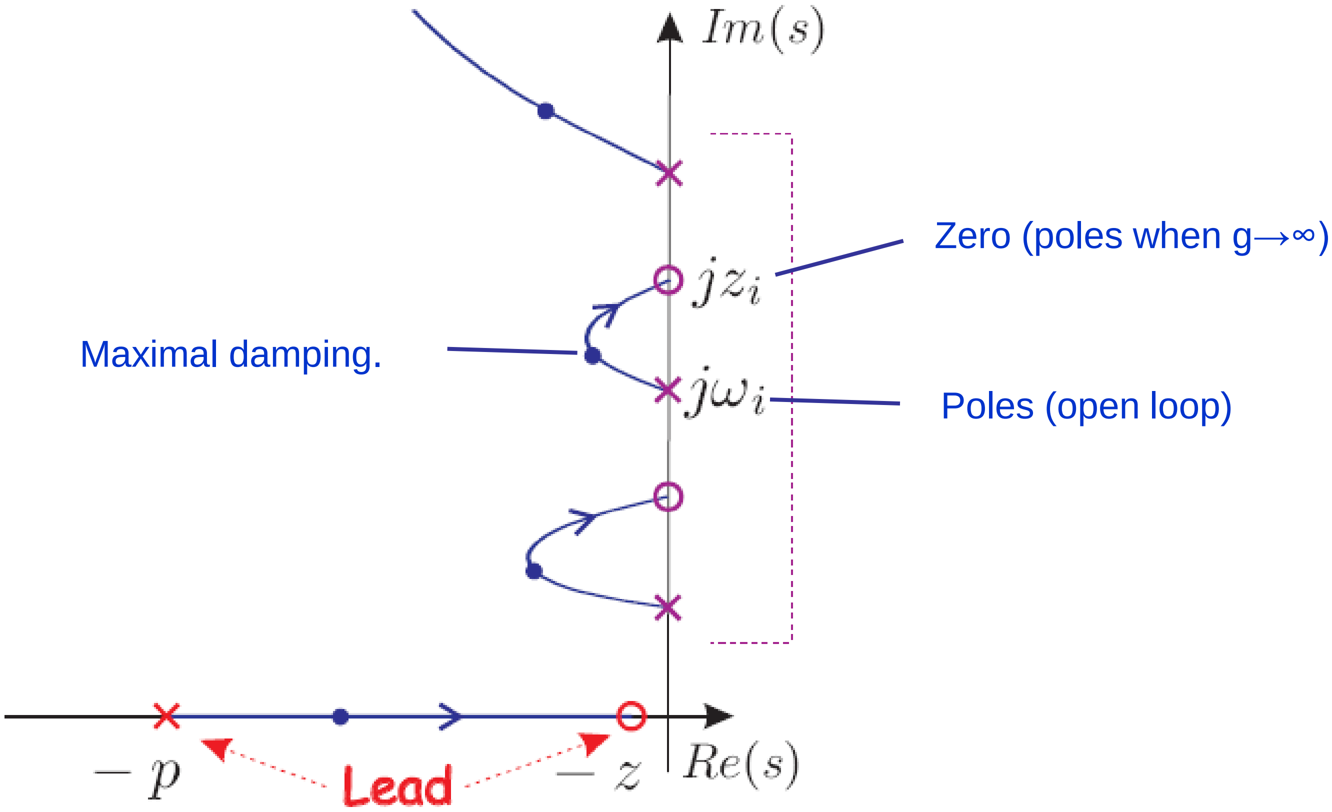
Conclusions from problem 2.4 :

1. The poles doesn't depend on the location of the actuators and the sensors (inherent property of the structure). Unlike the poles, the zeros are very sensitive.
2. The high frequency modes are of big influence on the position of the zeros. They can be neglected only if the quasi-static correction is considered.
3. A collocated sensor/actuator provides a pole zero alternane and thus a phase which remains always between +-180°.

P.5.12 Compare the transfer functions of Fig.3.26 from the point of view of control authority. Comment on the consequences of the error in predicting the zeros.



Typical root locus:



P.4.5 *Consider the PD regulator*

$$H(s) = g(Ts + 1)$$

applied to the open-loop structure

$$G(s) = \sum_{i=0}^{\infty} \frac{\phi_i(a)\phi_i(s)}{s^2 + \omega_i^2}$$

Assuming that the modes are well separated, show that, for small gain g, the closed-loop damping ratio of mode i is

$$\xi_i = gT \frac{\phi_i(a)\phi_i(s)}{2\omega_i}$$

Conclude on the stability condition (Gevarter).
[Hint: Use a perturbation method, $s = \omega_i[-\xi + j(1 + \delta)]$ in the vicinity of $j\omega_i$, and write the closed-loop characteristic equation.]

Solution:

1. The poles in closed loop $\mathbf{s_i^*}$ are given by solving:

$$1+H(\mathbf{s_i^*})G(\mathbf{s_i^*})=0 \qquad (i=1,..,n)$$

2. Because the gain is small, the closed loop poles $\mathbf{s^*}$ are very close to the poles in open loop, so : $\mathbf{s_i^*} \sim j\omega_i$. The response of the system at $\mathbf{s_i^*}$ is thus dominated by the response of mode i :

$$G(\mathbf{s_i^*}) = \sum_{k=1}^{n} \frac{\phi_k(a)\phi_k(s)}{\mathbf{s_i^{*2}} + \omega_k^2} \approx \frac{\phi_i(a)\phi_i(s)}{\mathbf{s_i^{*2}} + \omega_i^2}$$

3. It is easyer to assume as advised :

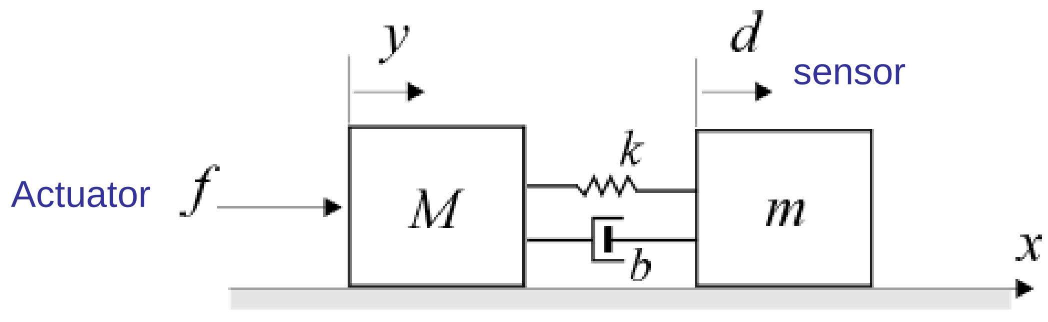
$$\mathbf{s_i^*} = \omega_i\left(-\xi + j(1+\delta)\right)$$

With ξ and δ to be found.

$\Rightarrow$ Stability condition : $\phi_i(a)\,\phi_i(s) > 0$

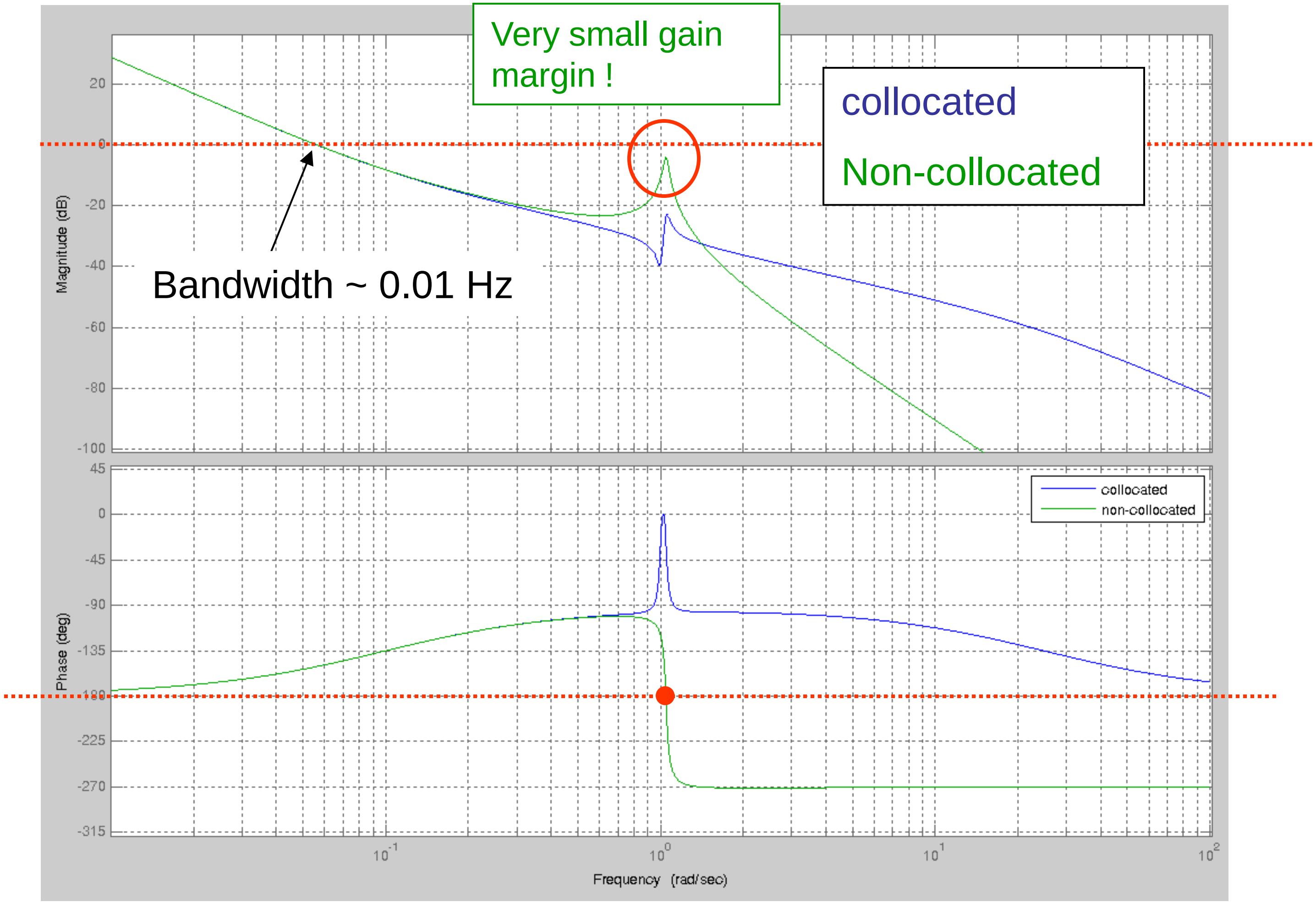
P.4.6 *Consider a simply supported uniform beam with a point force actuator and a displacement sensor. Based on the result of the previous problem, sketch a non-collocated actuator and sensor configuration such that a PD regulator is stabilizing for the first three modes.*

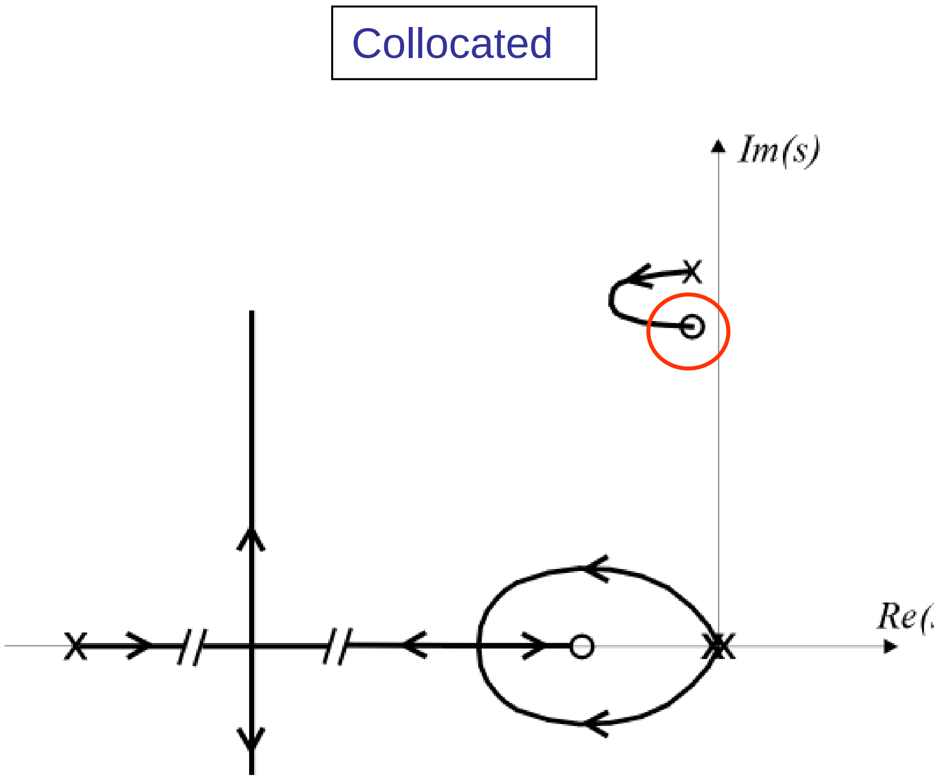
P.4.1 *Consider the lead compensator for the non-collocated control of the two-mass system (section 4.4). Determine the value of the damping ratio ξ which would reduce the gain margin to zero. What would be the gain margin if $\xi = 0.04$ instead of $\xi = 0.02$.*

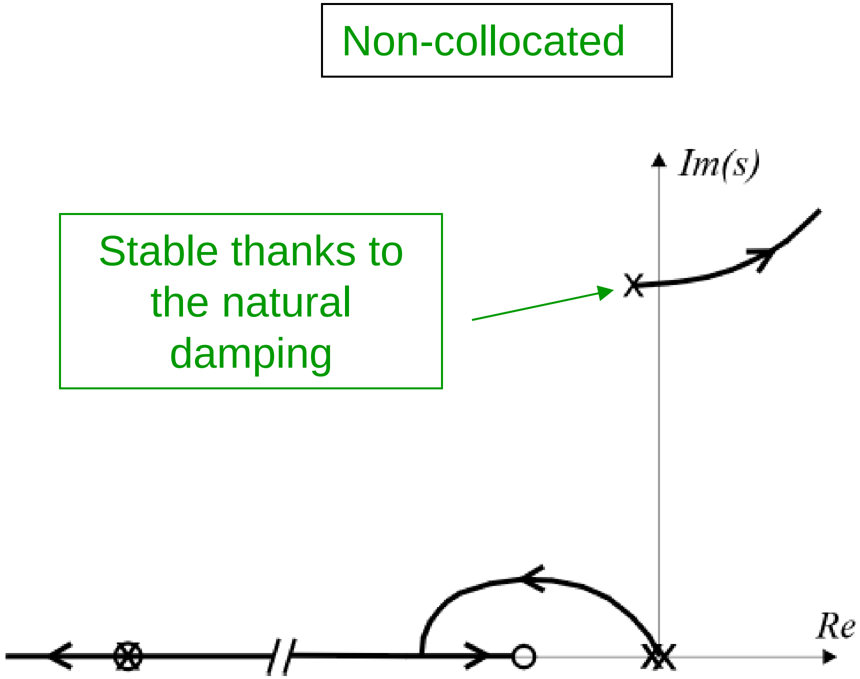


$$G_2(s) = \frac{D(s)}{F(s)} = \frac{2\xi\omega_0 s + \omega_0^2}{Ms^2\left[s^2 + (1+\mu)\left(2\xi\omega_o s + \omega_o^2\right)\right]} \qquad (4.7)$$

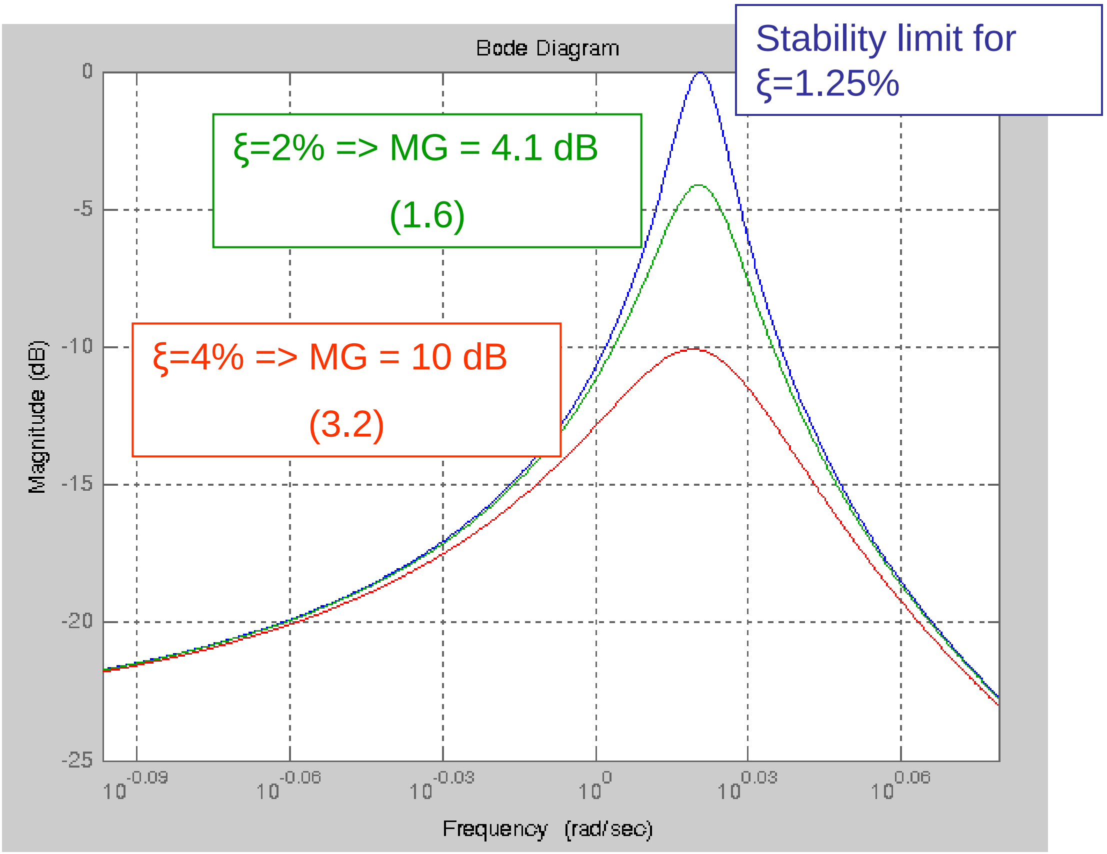
$$H(s) = g\frac{Ts+1}{\alpha Ts+1} \qquad (\alpha < 1)$$


Very small gain margin !
collocated
Non-collocated
Bandwidth ~ 0.01 Hz
Magnitude (dB)
Phase (deg)
collocated
non-collocated
Frequency (rad/sec)


Collocated
Im(s)
Re(

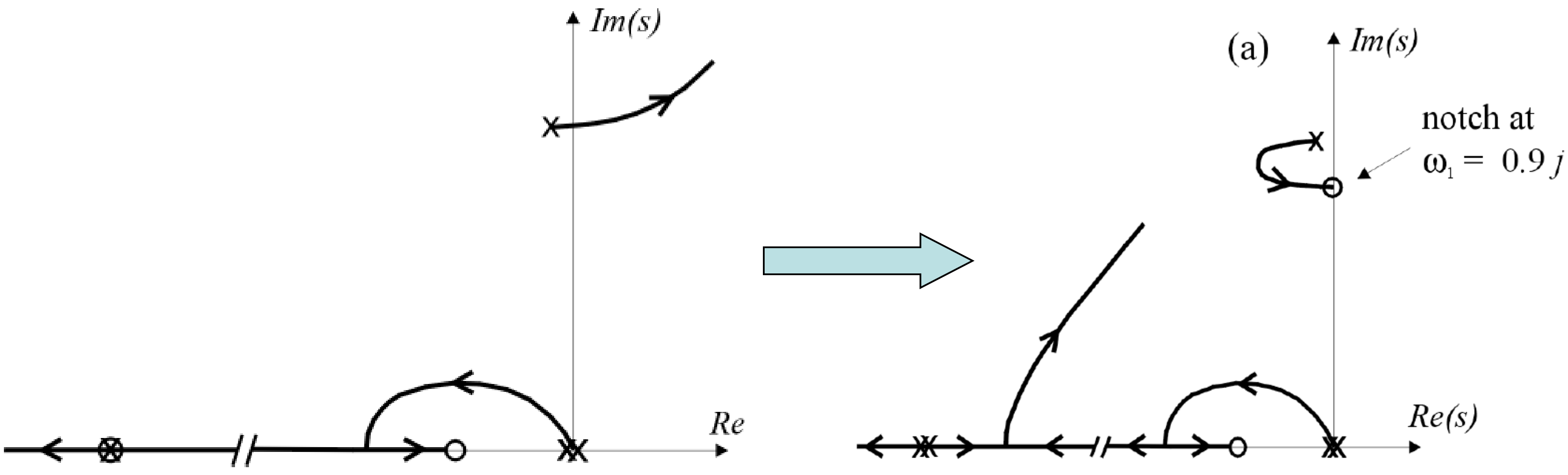

Non-collocated
Stable thanks to the natural damping
Im(s)
Re

Non-collocated control

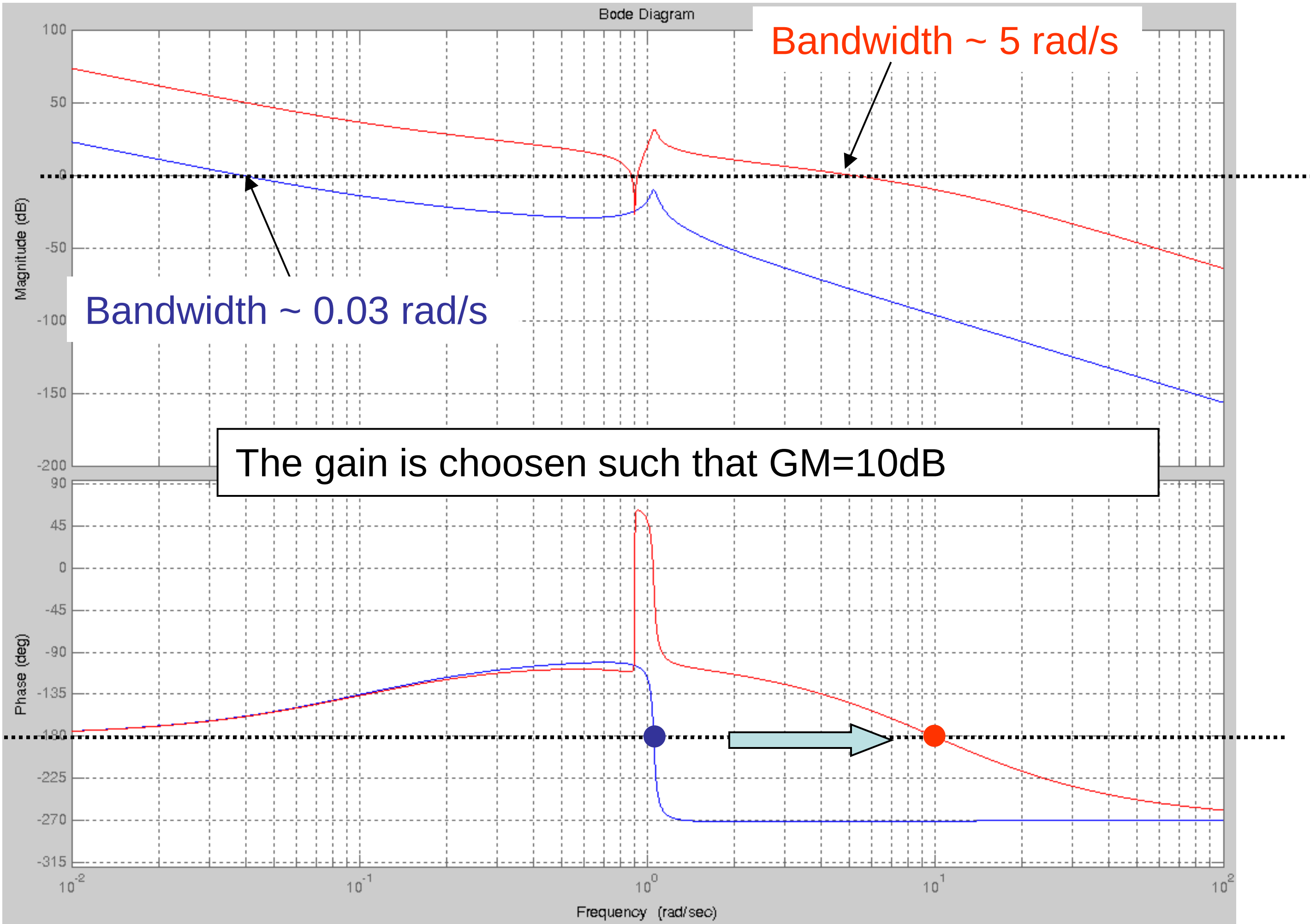


P.4.2 *Consider the lead compensator plus notch filter (4.10) for the non-collocated control of the two-mass system (section 4.5). Draw the corresponding Bode plots. Select a reasonable value of the gain g and compare the bandwidth, the gain and phase margins with those of the lead compensator of Fig.4.6.*

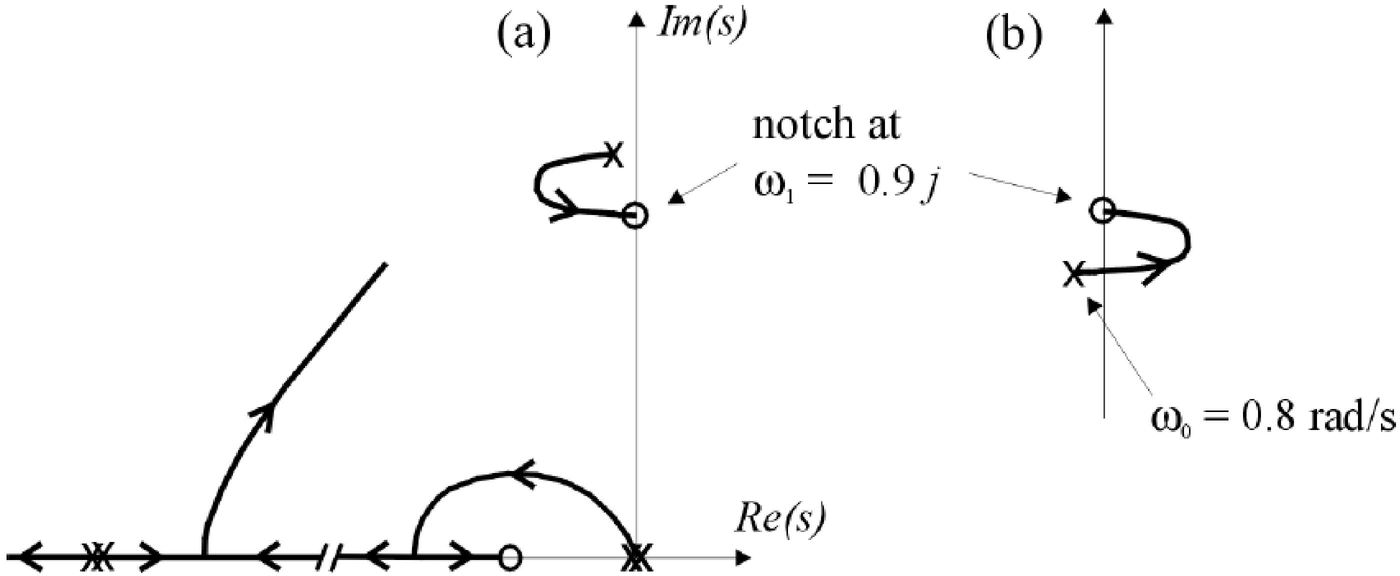
« notch » = zero at a given frequency, such that it simplifies a pole wich can become unstable.



$$H(s) = g \cdot \frac{Ts+1}{\alpha Ts+1} \cdot \boxed{\frac{s^2/\omega_1^2+1}{(s/a+1)^2}}$$



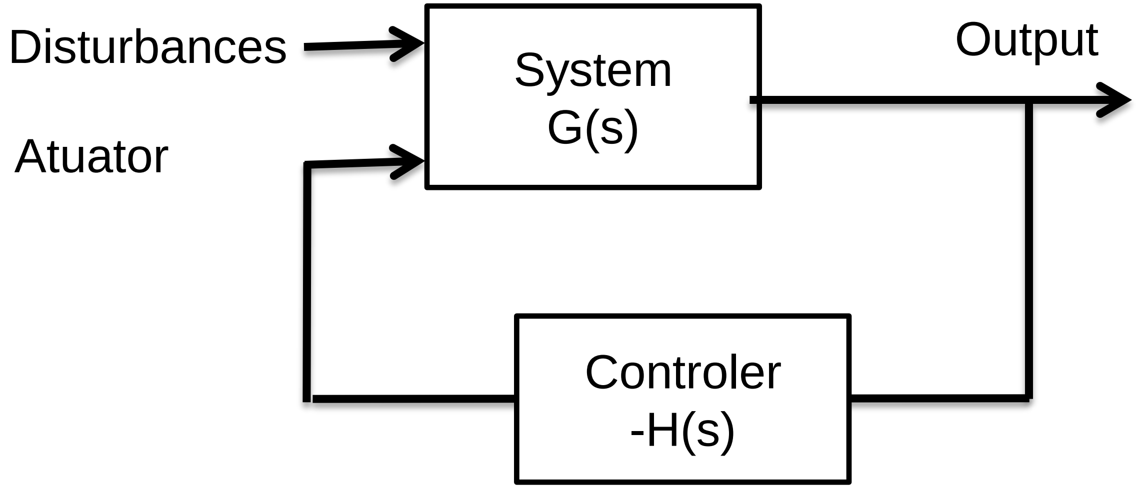
P.4.3 *(a) Repeat the previous problem when the frequency of the appendage is lower than that of the notch filter (ω_0 =0.8 rad/sec); compare the Bode plots and comment on the role of the damping.*

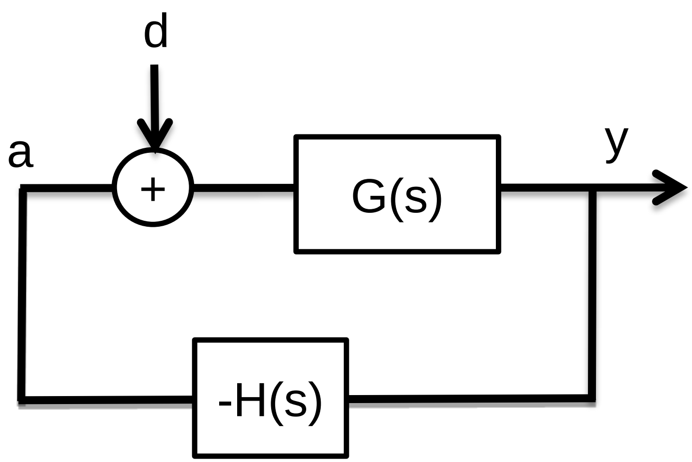


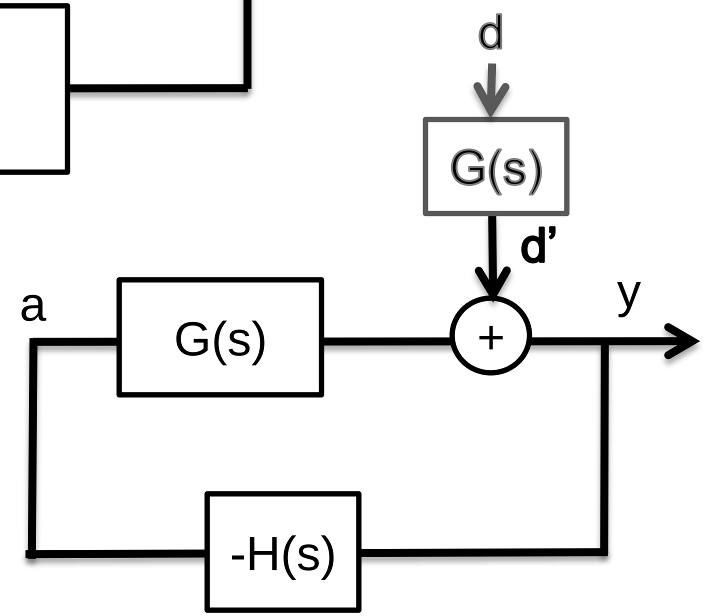
Conclusions from P.4.3 :

•Be carrefull when a « pole-zero flipping » can happen.

• It is very difficult to control a structure when it is lightely damped and non-collocated/







Disturbance at the input
(physical representation)

$y = G/(1+GH)d$

Disturbance at the output
(Analitical representation)

$y = \mathbf{1/(1+GH)}d' = G/(1+GH)d$