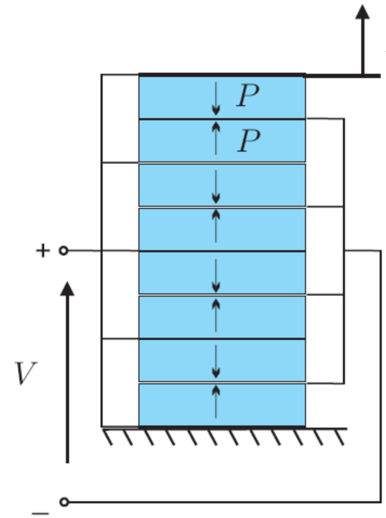
Lesson 2 :

Sensors and Actuators

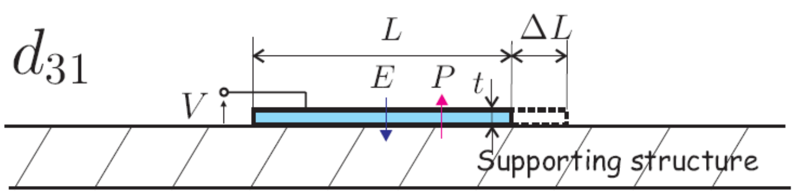
Piezoelectric actuator

d_{33}



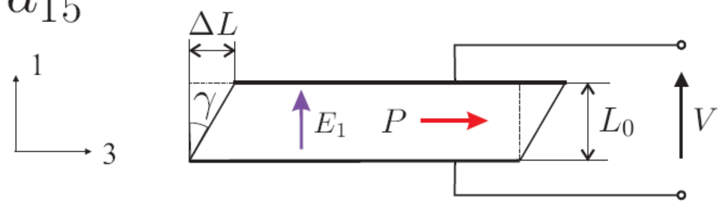
$$\Delta L = n d_{33} V$$

d_{31}



$$\Delta L = E d_{31} L$$
$$E = V/t$$

d_{15}



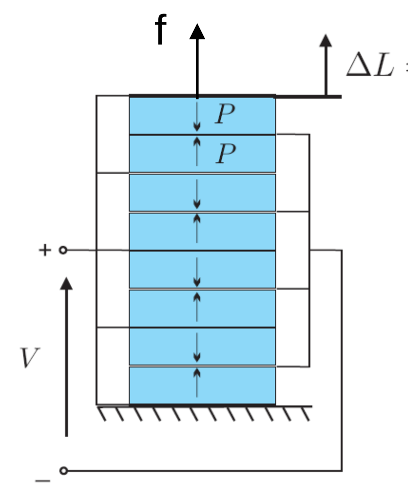
$$\gamma = d_{15} E_1$$
$$\Delta L = \gamma L_0$$

Problem P.3.10

Consider a one dimensional piezoelectric element (active strut). Show that the stiffness with open-electrodes K_a^* and the stiffness with short-circuited electrode, K_a, are related by

$$K_a^* = \frac{K_a}{1-k^2}$$

where k is the electromechanical coupling factor.



Unidirectional material equation: p100

$$\begin{Bmatrix} D \\ S \end{Bmatrix} = \begin{bmatrix} \varepsilon^T & d_{33} \\ d_{33} & s^E \end{bmatrix} \begin{Bmatrix} E \\ T \end{Bmatrix}$$

Cross section: A
Thickness: t
# of disks in the stack: n
$l = nt$

Electric charge:
$Q = nAD$

Capacitance:
$C = n^2 \varepsilon A / l$

1st step : Integer the equations to obtain « macroscopiques » values:

- Q = electrical charge. Q=nAD
- Δ = total strain. Δ =SL
- V = Voltage between the electrodes. E=V/T=nV/L
- f = Axial force applied to the stack. F=AT.

Finaly, we get:

$$\begin{Bmatrix} Q \\ \Delta \end{Bmatrix} = \begin{bmatrix} \frac{\varepsilon^T A n^2}{l} & n d_{33} \\ n d_{33} & \frac{s^E l}{A} \end{bmatrix} \begin{Bmatrix} V \\ f \end{Bmatrix}$$

C

K_a

$$\Rightarrow \quad K_a = \left.\frac{f}{\Delta}\right|_{V=0} = \frac{A}{s^E l}$$

2nd step: we do the same for the coupling factor equation:

$$k^2 = \frac{{d_{33}}^2}{s^E \varepsilon^T} = \frac{n^2 {d_{33}}^2 K_a}{C}$$

3rd step: we invert the constitutive equation :

$$\begin{Bmatrix} V \\ f \end{Bmatrix} = \frac{K_a}{C(1-k^2)} \begin{bmatrix} 1/K_a & -nd_{33} \\ -nd_{33} & C \end{bmatrix} \begin{Bmatrix} Q \\ \Delta \end{Bmatrix}$$

The stiffness in open circuit (Q=0) is so:

$$\left.\frac{f}{\Delta}\right|_{Q=0} = \frac{K_a}{(1-k^2)}$$

Because of the electromechanical coupling, the boundary conditions has a big effect on the piezo behavior!!

Remark 1:

$$Q = C\,V + nd_{33}\,f$$

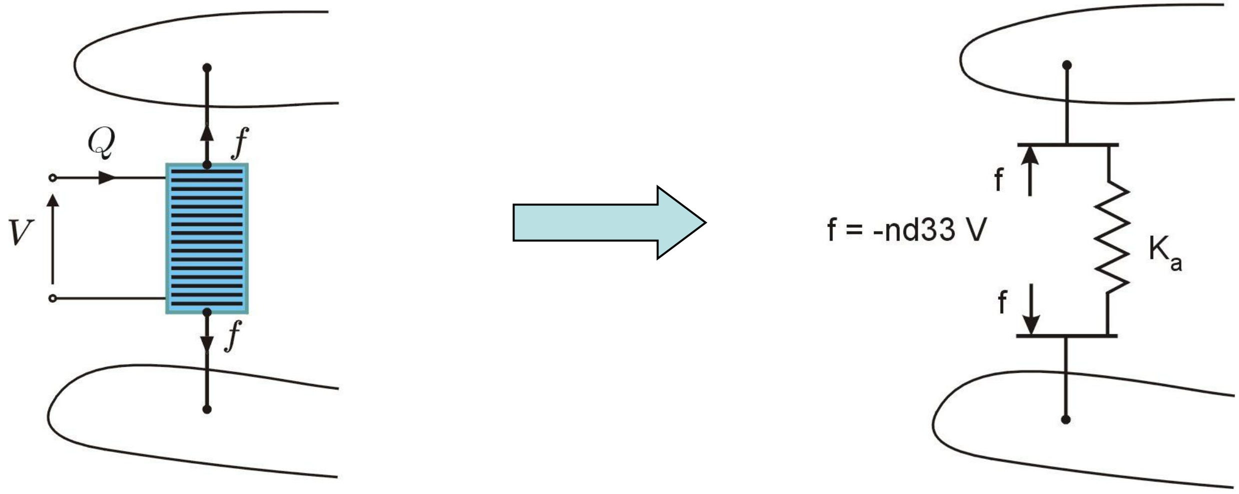
Use as « force sensor » : if the piezo is short-circuited (V=0), the generated electrical charge is proportional to the axial force *f*.

Remark 2:

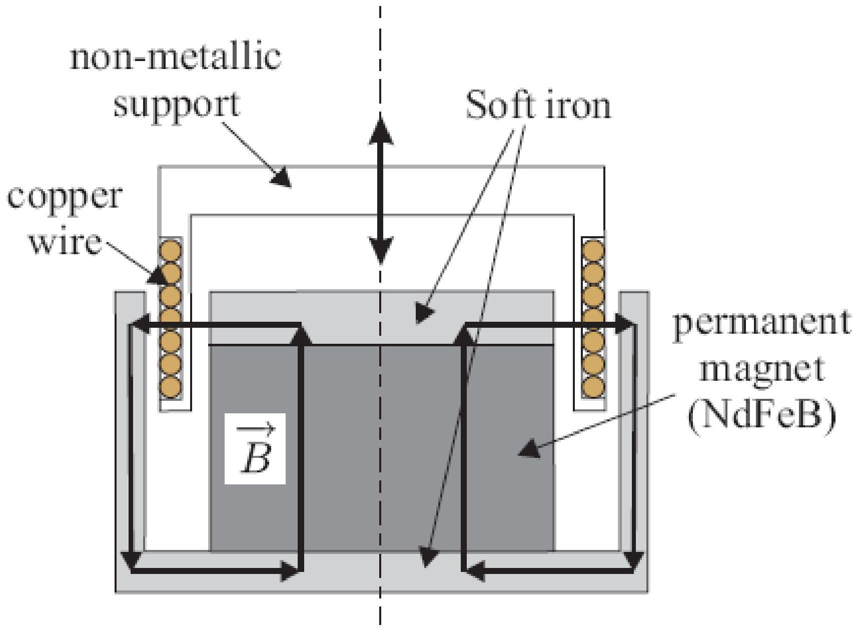
$$f = K_a(\Delta - n d_{33} V)$$

I.e. the applied force by the piezo is composed by an elastic force due to its stiffness and a force proportional to the applied voltage *V*.

Thus, a mechanical structure equiped with a piezoelectric transducer working in d_{33} mode can be modeled easily !



Electromagnetic transducers

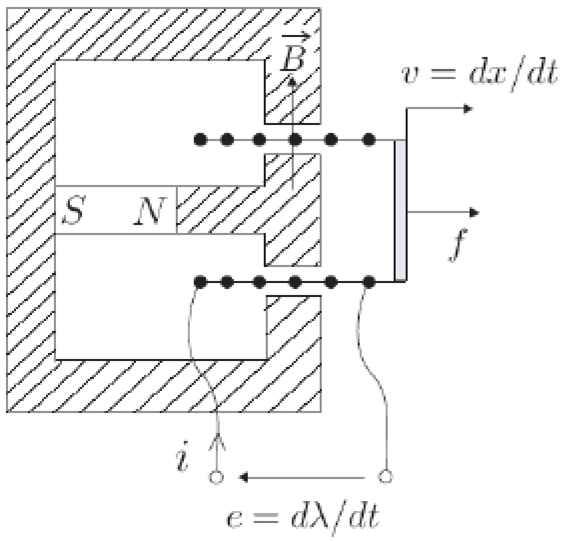


$$V = T_v \; \Delta\dot{x}$$

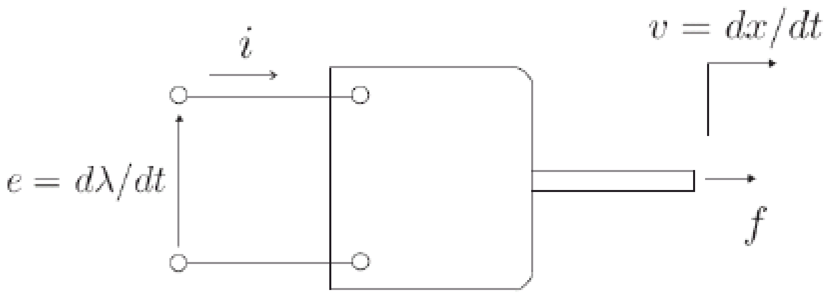
$$f_c = T_v \; I$$

$$T_v \; = \; 2\pi n r B$$

Voice-coil transducer



Symbolic representation:



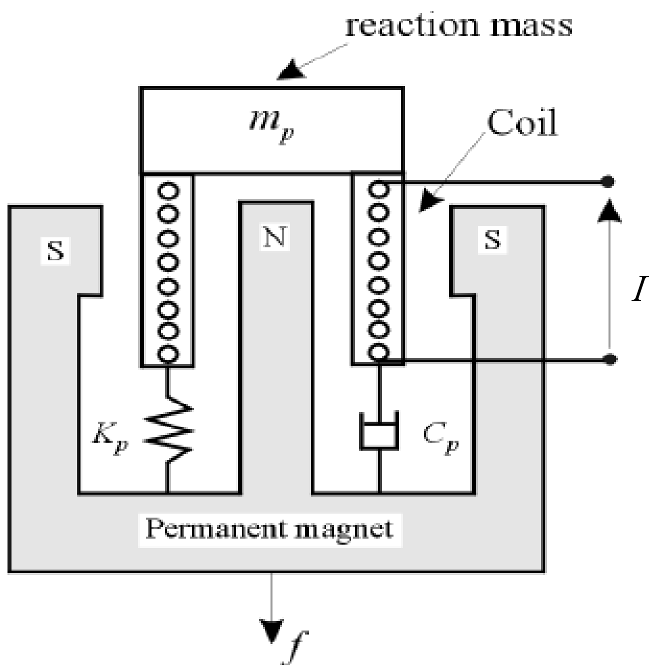
$$e = Tv \quad \text{(Faraday)}$$
$$-Ti = f \quad \text{(Lorentz)}$$

T = transducer constant
(in volt.sec/m or N/Amp)

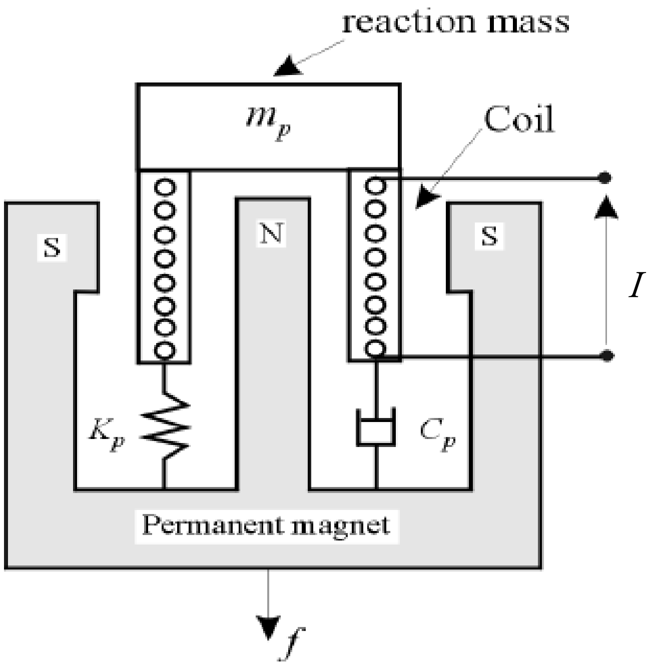
Power: $$ei + fv = Tvi - Tiv = 0$$

(perfect transformer; energy cannot be stored)

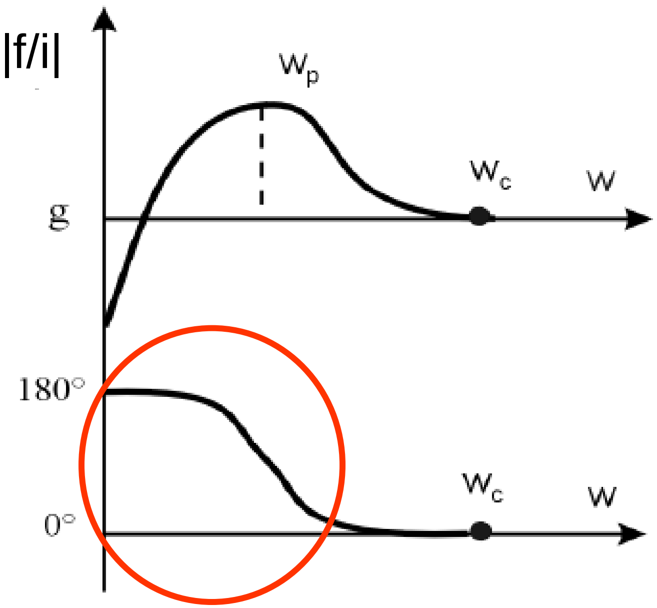
Inertial actuator (« Proof-Mass »)



Inertial actuator (« Proof-Mass »)



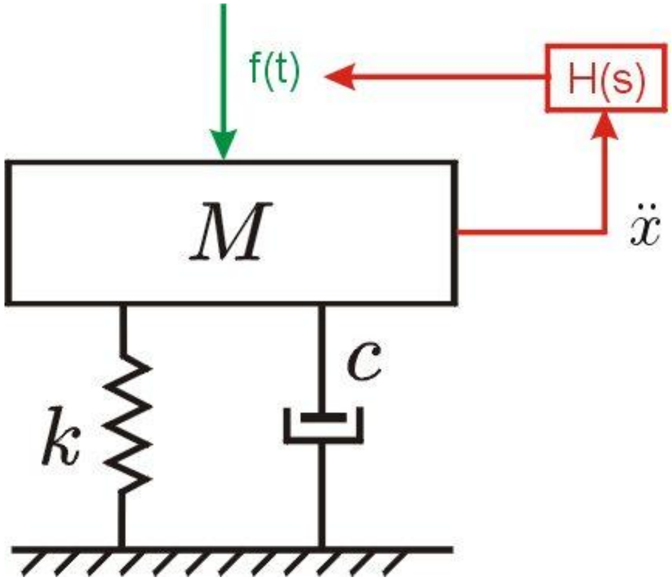
$$f/i = g\frac{s^2}{s^2 + 2\xi_p\omega_p s + \omega_p^2}$$



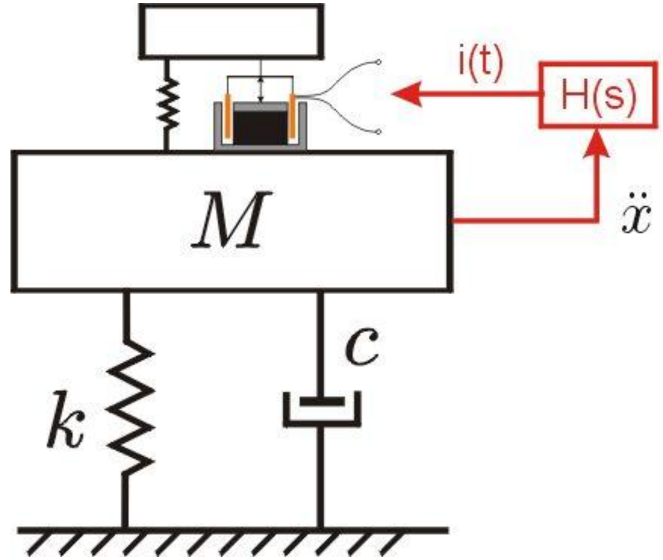
The actuator is not ideal bellow ω_c!

P 5.5 *Consider a s.d.o.f. oscillator controlled with a proof-mass actuator; analyse the effect of the actuator dynamics on the active damping by acceleration feedback with a second order filter. Show that if the natural frequency of the actuator is much lower than that of the filter* ($\omega_p \ll \omega_f$)*, the actuator dynamics has little influence on the root locus for small gains, but that the system becomes unstable for large gains.*

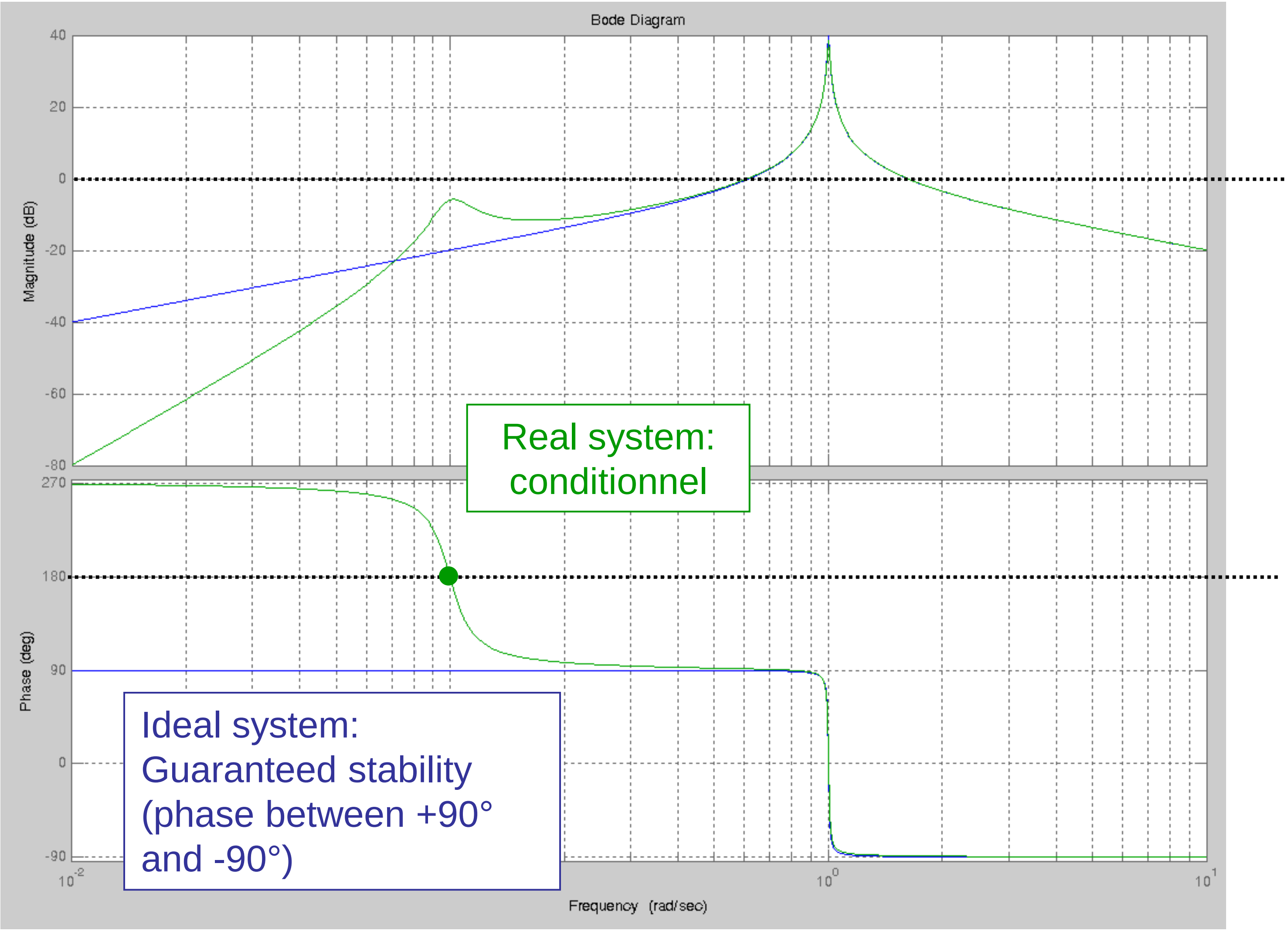
Ideal case:
Guaranteed
stability



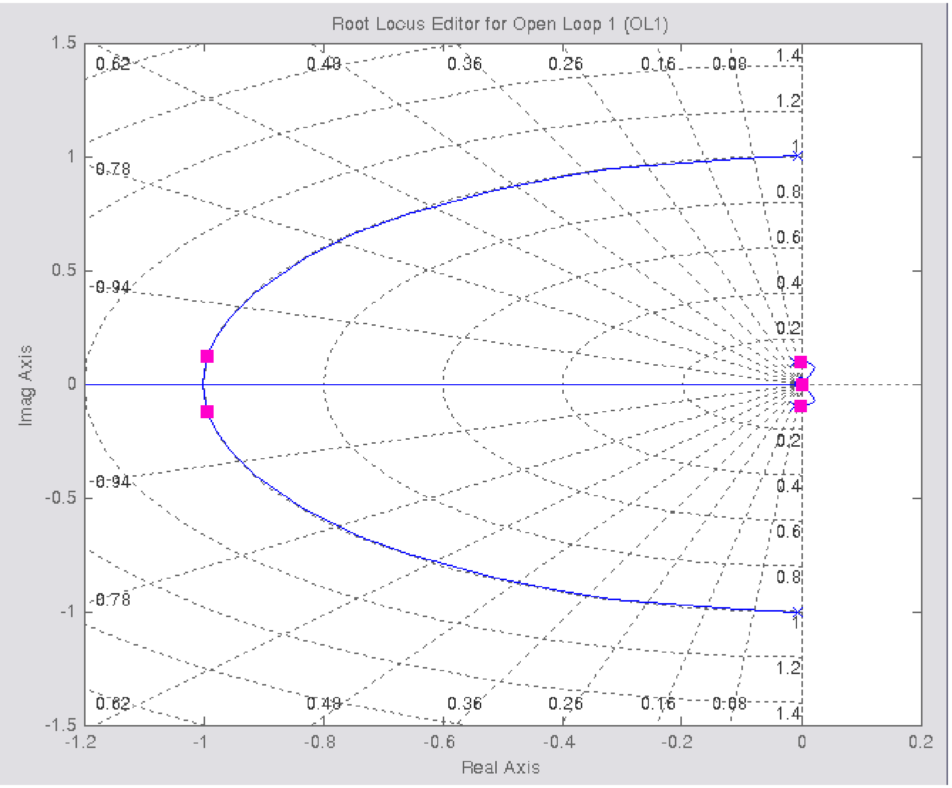
In reality



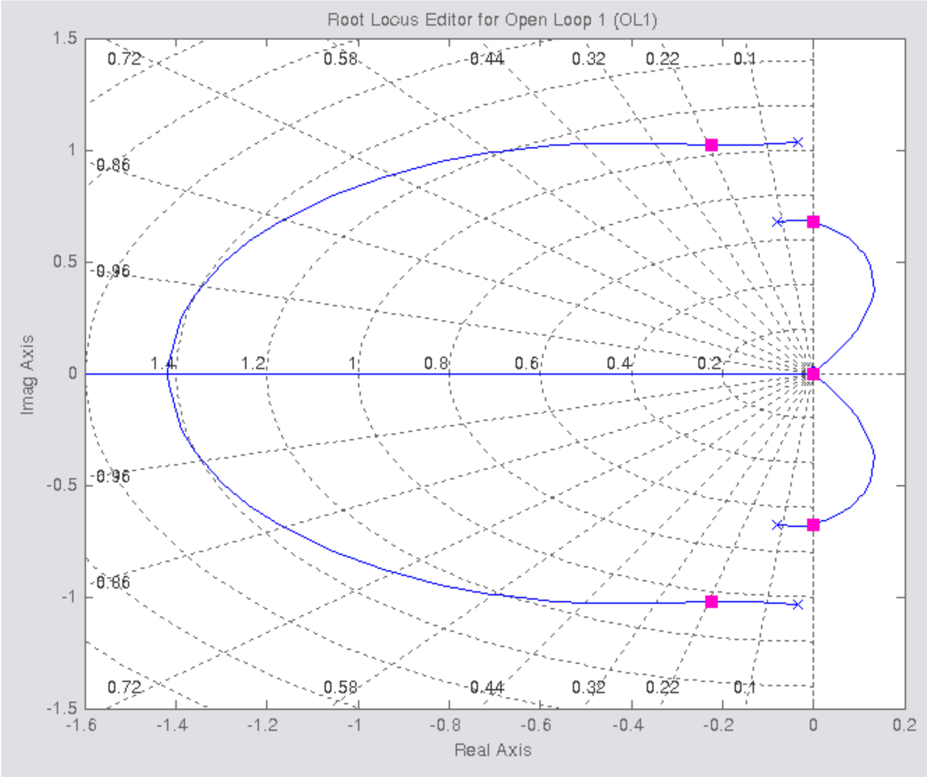
Acceleration feedback, « derivative » controler:



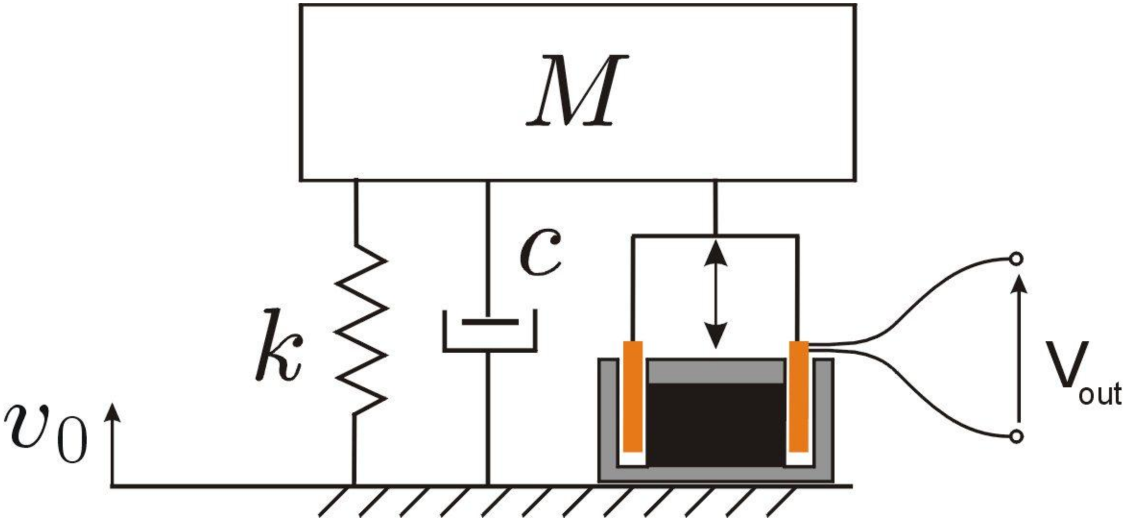
$\omega_f <<< \omega_p$



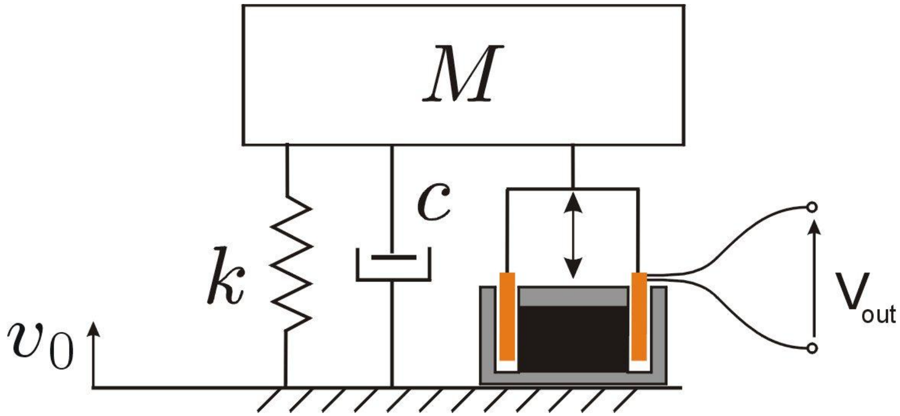
$\omega_f \sim= \omega_p$



Seismic sensor (geophone)



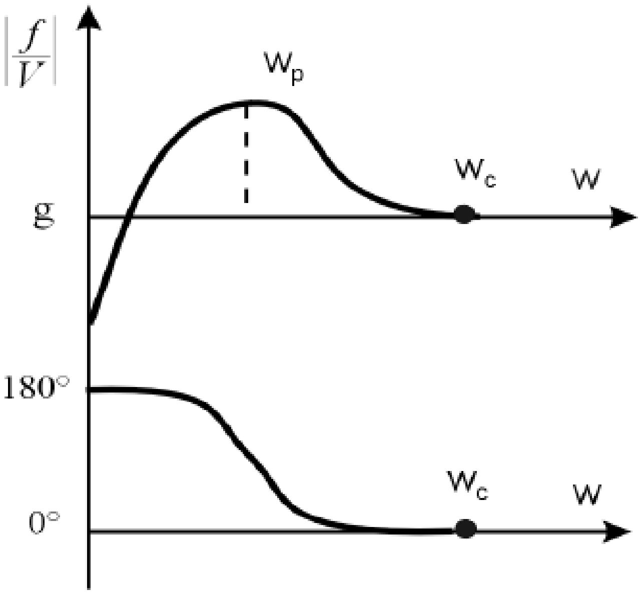
1. Sensor equation



$$X(s) = \frac{cs + K}{Ms^2 + cs + K} X_0(s)$$

$$V_{out} = T_v s\,(X - X_0)$$

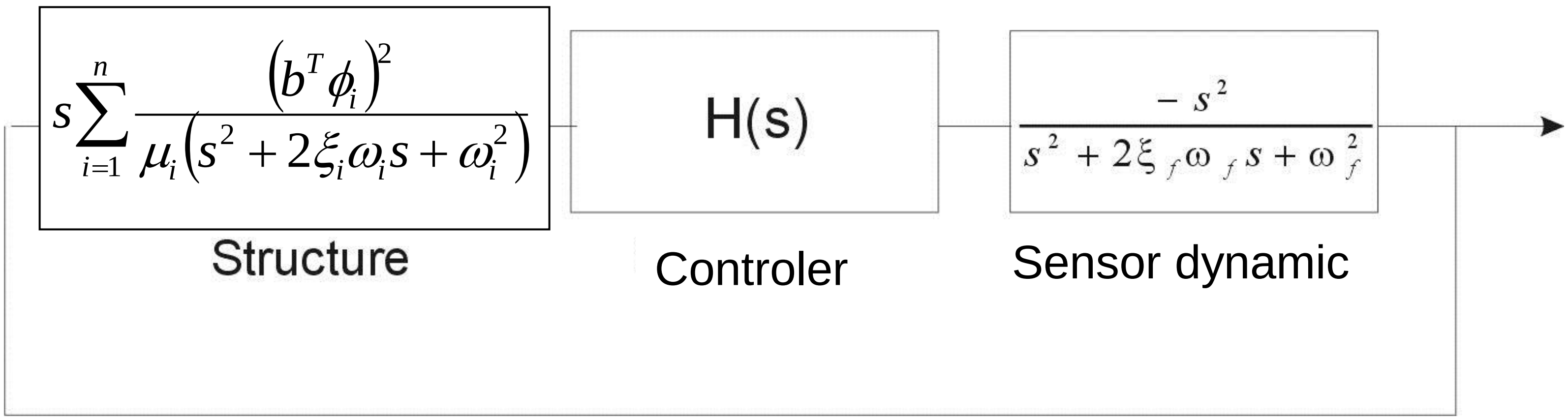
$$\frac{V_{out}}{sX_0} = T_v \frac{s^2}{s^2 + C/Ms + K/M}$$



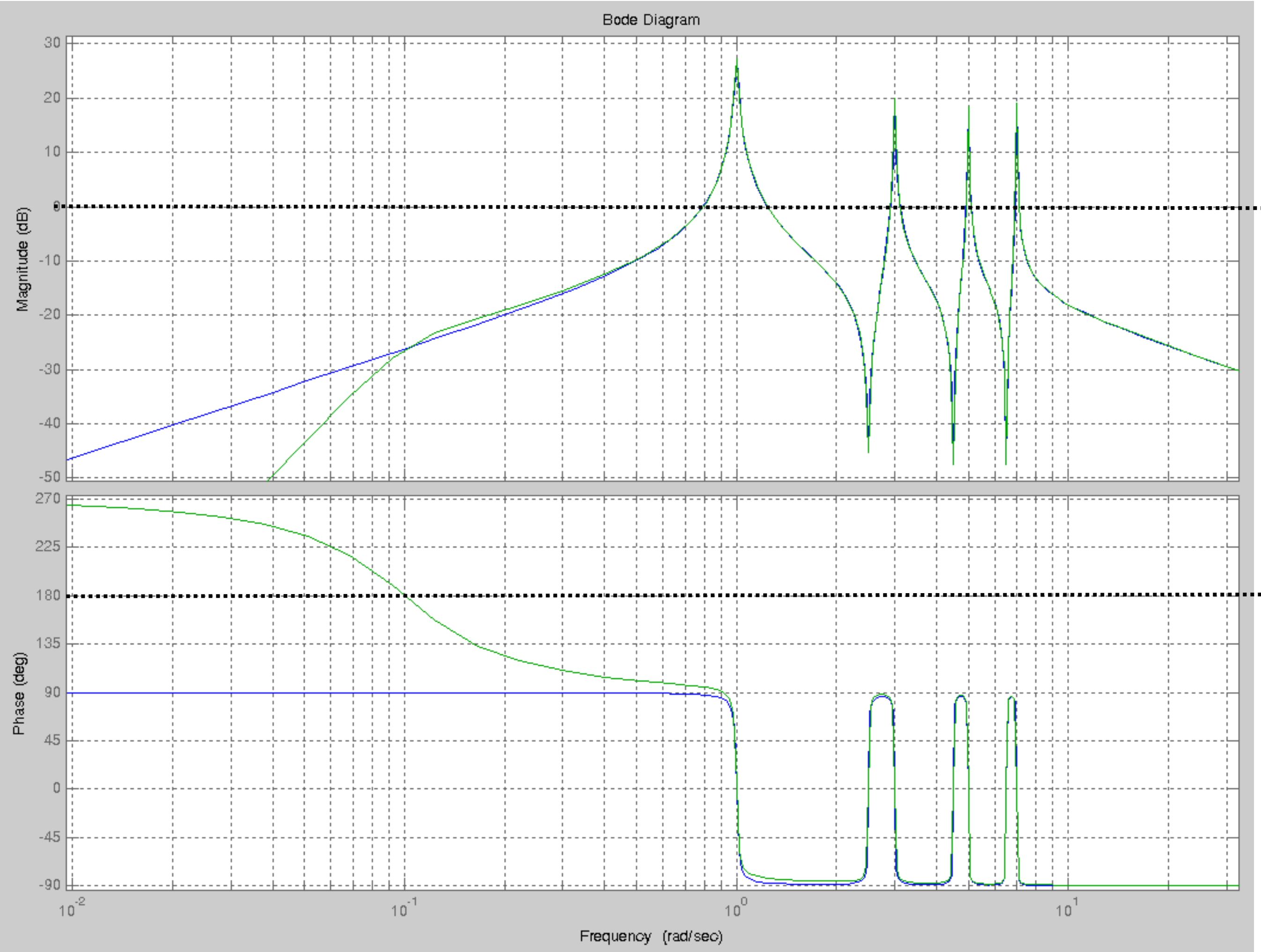
2. Control system

Hypothesis : the sensor mass is <<< structure mass

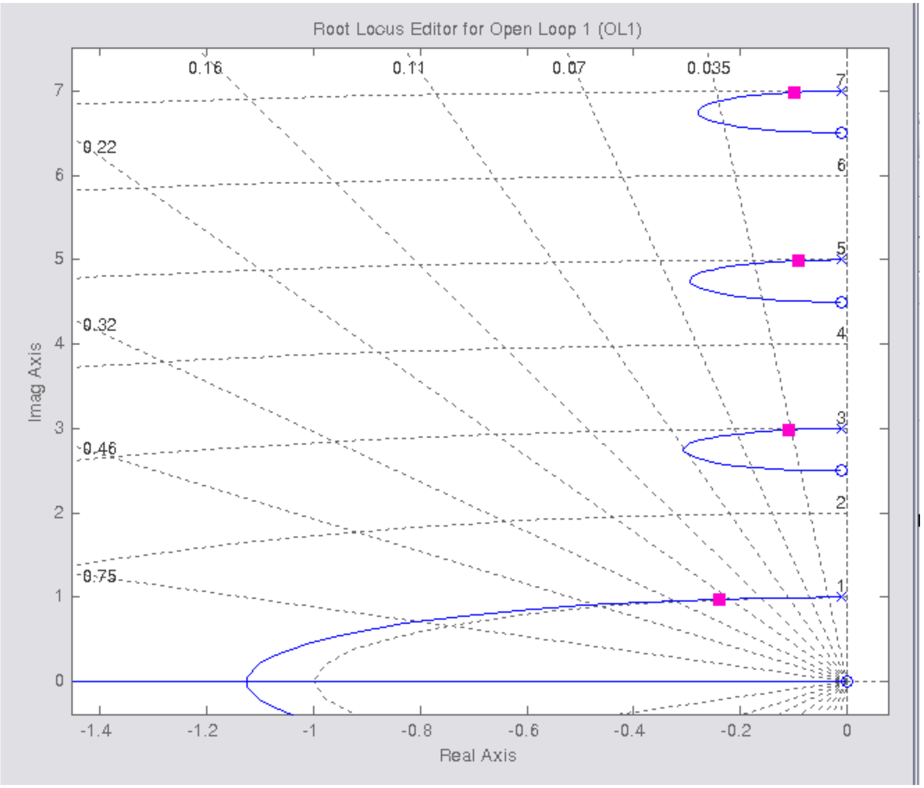
=> Adding the sensor doesn't affect G(s).



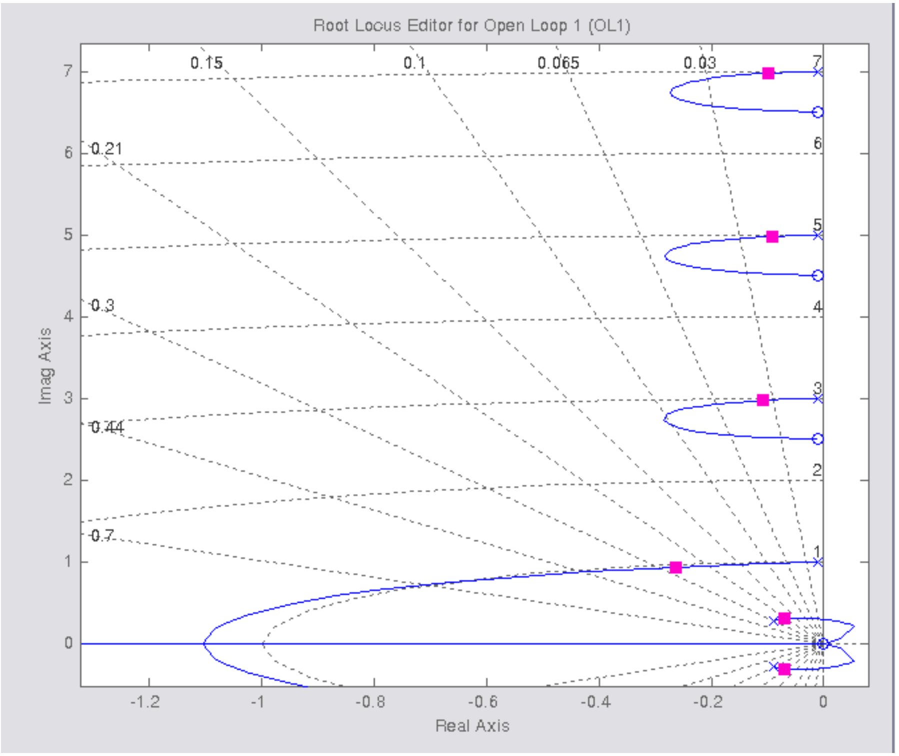
3. Matlab results



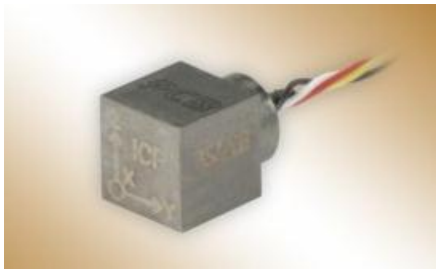
Ideal case (guaranteed stability):

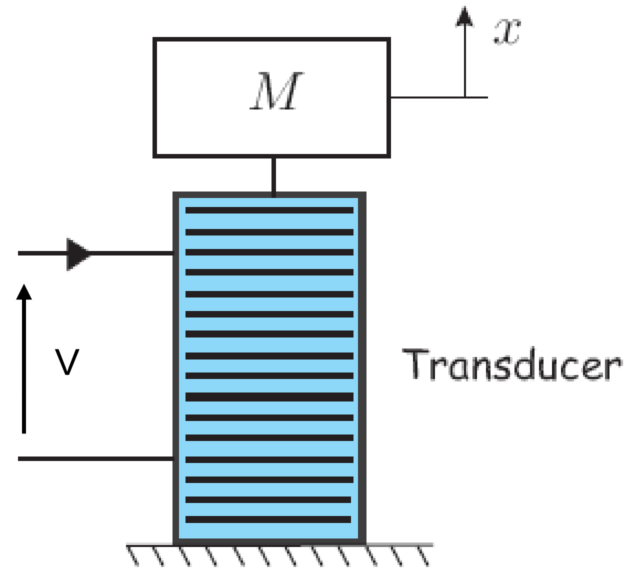


In reality



Piezoelectric accelerometer





Generated electrical charge

$$\begin{Bmatrix} Q \\ \Delta \end{Bmatrix} = \begin{bmatrix} C & nd_{33} \\ nd_{33} & 1/K_a \end{bmatrix} \begin{Bmatrix} V \\ f \end{Bmatrix}$$

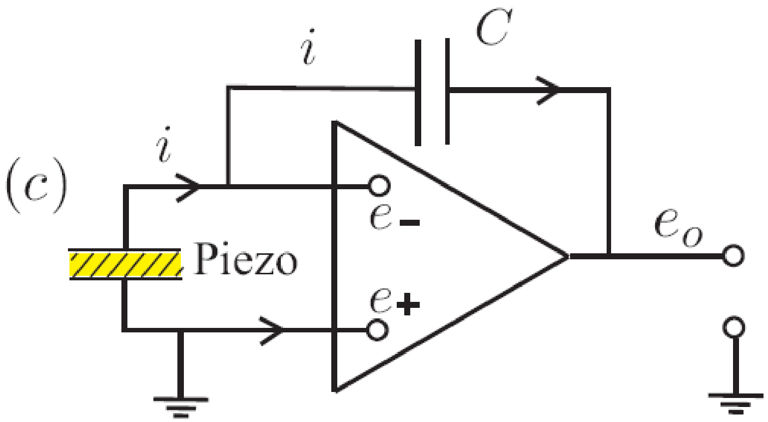
Axial force

We have:

1. $f = Ms^2x$
2. $Q = nd_{33}f$ (si V=0!)

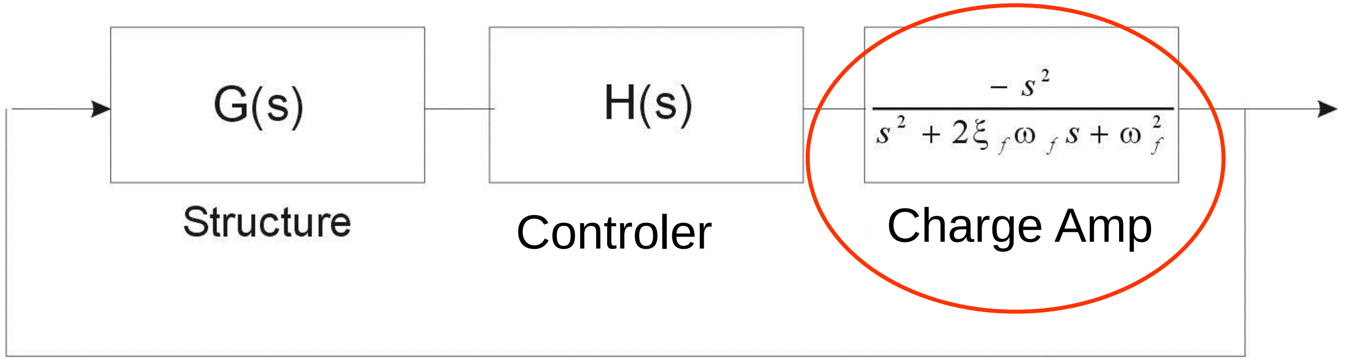
=> Q is a measure of s^2x.

In practice:



The charge amplifier introduces a high-pass filter and invert the sign

$$V(s) = \frac{-s^2}{s^2 + 2\xi_f \omega_f s + \omega_f^2} Q(s)$$



P 5.4 *Consider a vibrating structure with a point force actuator collocated with an accelerometer. For the two compensators discussed in connection with the acceleration feedback, analyse the effect of the charge amplifier dynamics [Equ.(3.41)] on the root locus. Show that the stability is no longer guaranteed, but if* $\omega_c \ll \omega_1$*, the sensor dynamics does not affect the control system for small gains.*