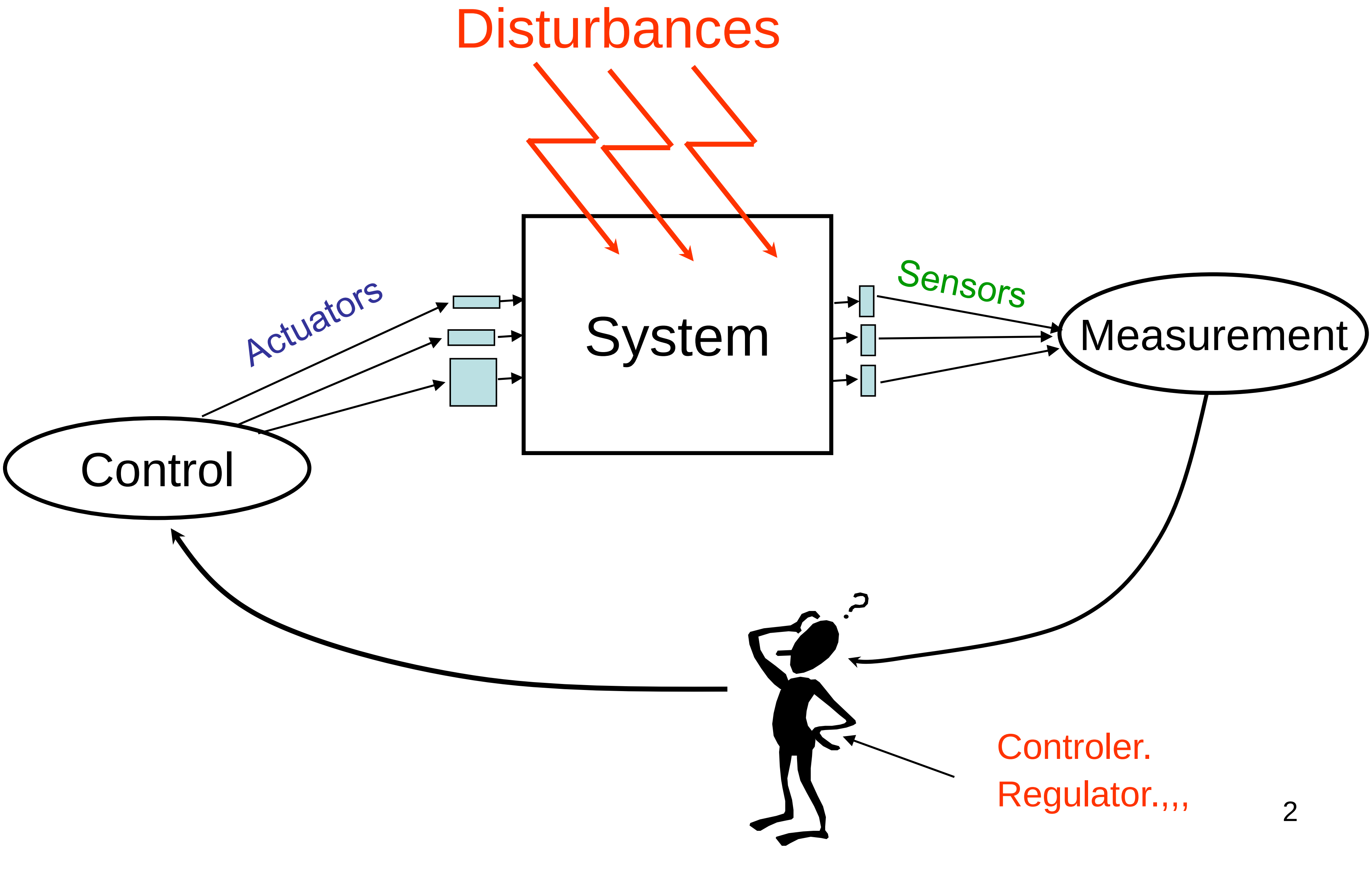
Active Control ?

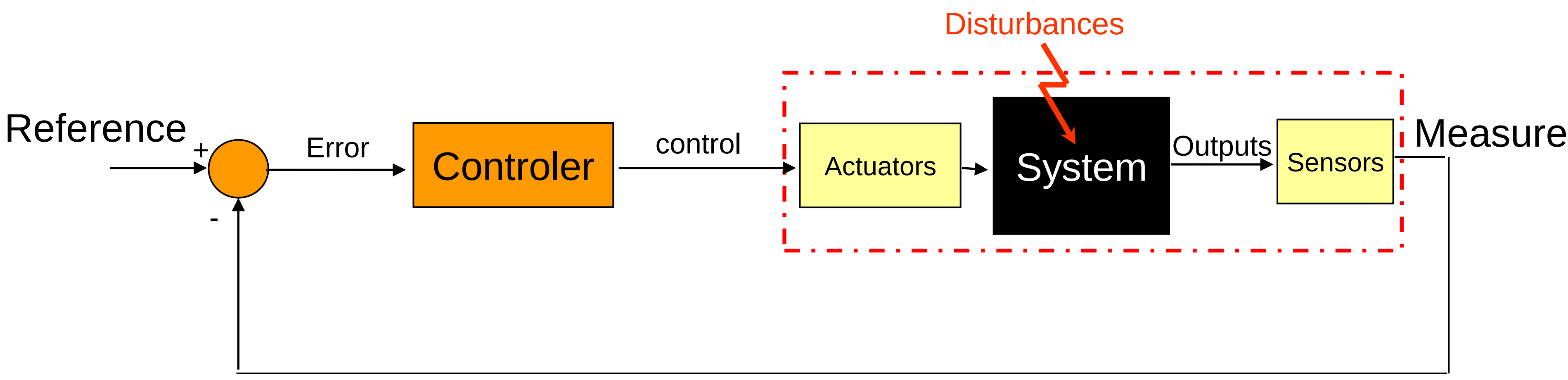
Contact : bmokrani@ulb.ac.be

Website : http://scmero.ulb.ac.be → Teaching

Active Control ?



Aims of an Active Control



- **Stabilisation** :

    Inverted pendulum, Launcher,…etc.

- **Control and regulation :**

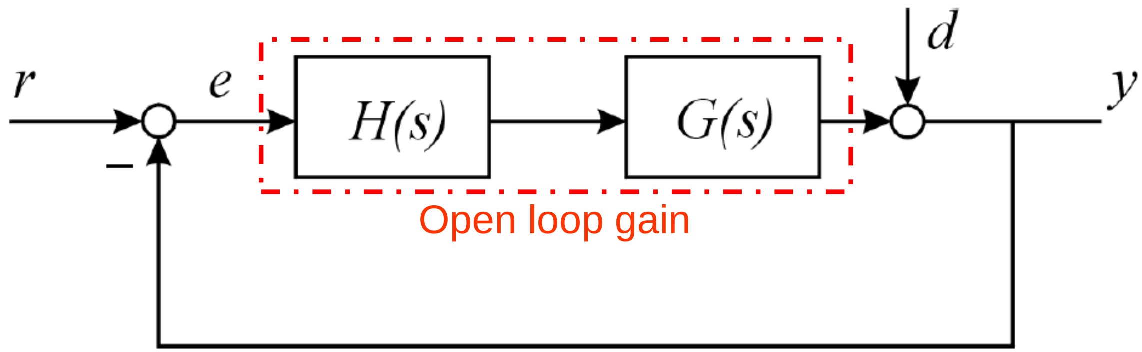
    Temperature control of a room, robotics,…etc.

- **Disturbance rejection:**

    Vibration isolation of a lithography table…

The 3 objectives are coupled

Analysis and controler synthesis



- **Stability:**
  - Gain margin and phase margin Open Loop : Nyquist, Nichols or Bode diagrams.
  - Closed Loop system poles (Pole Map).

- **Perturbation rejection = following the reference** :

Sensitivity function : $$\frac{y}{d} = \frac{1}{1+GH}$$ Maximize *GH*

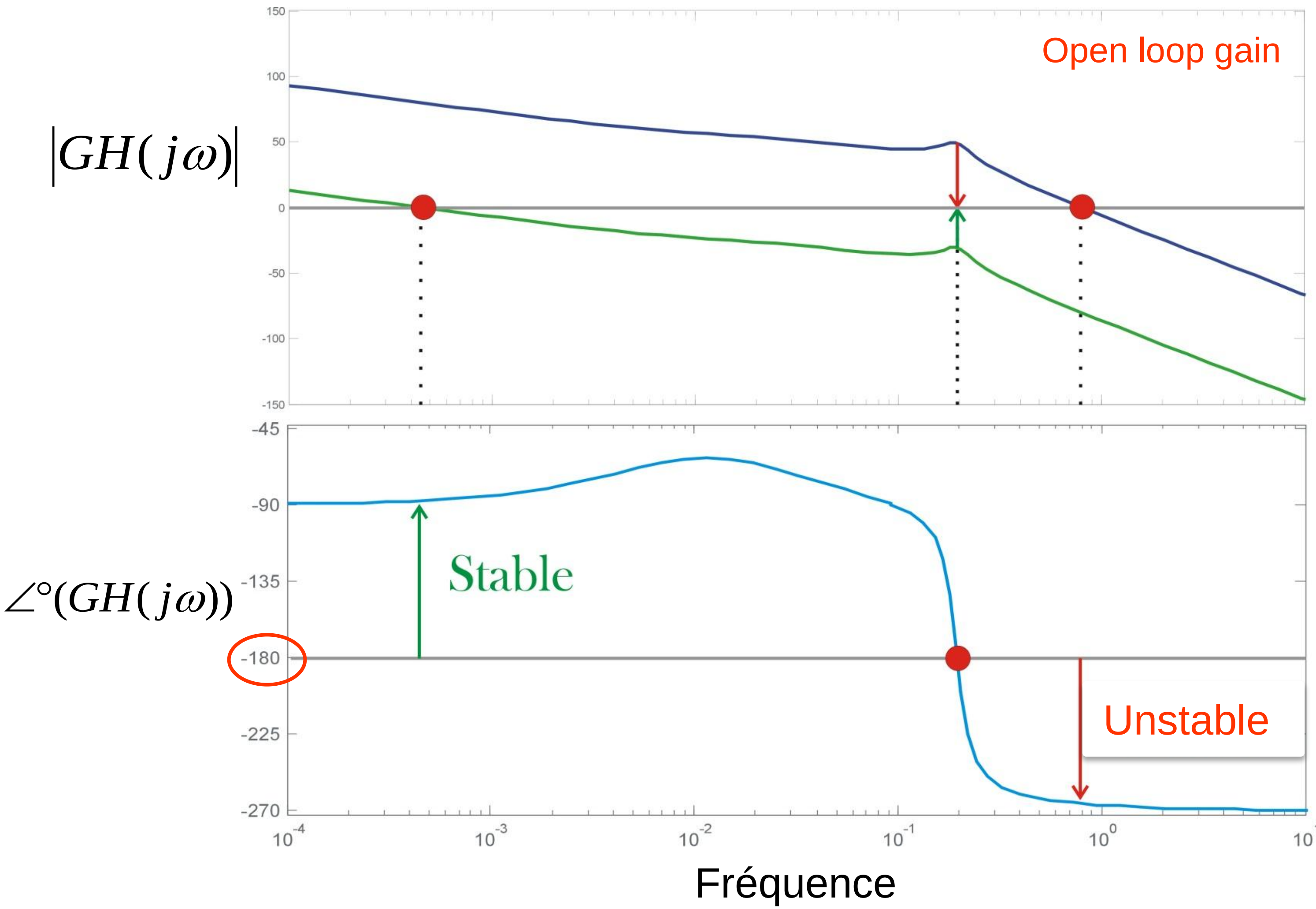
Complementary Sensitivity function : $$\frac{y}{r} = \frac{GH}{1+GH}$$ Maximize *GH*

Stability requierements limit achievable performances !

Bode Diagram : performance specification

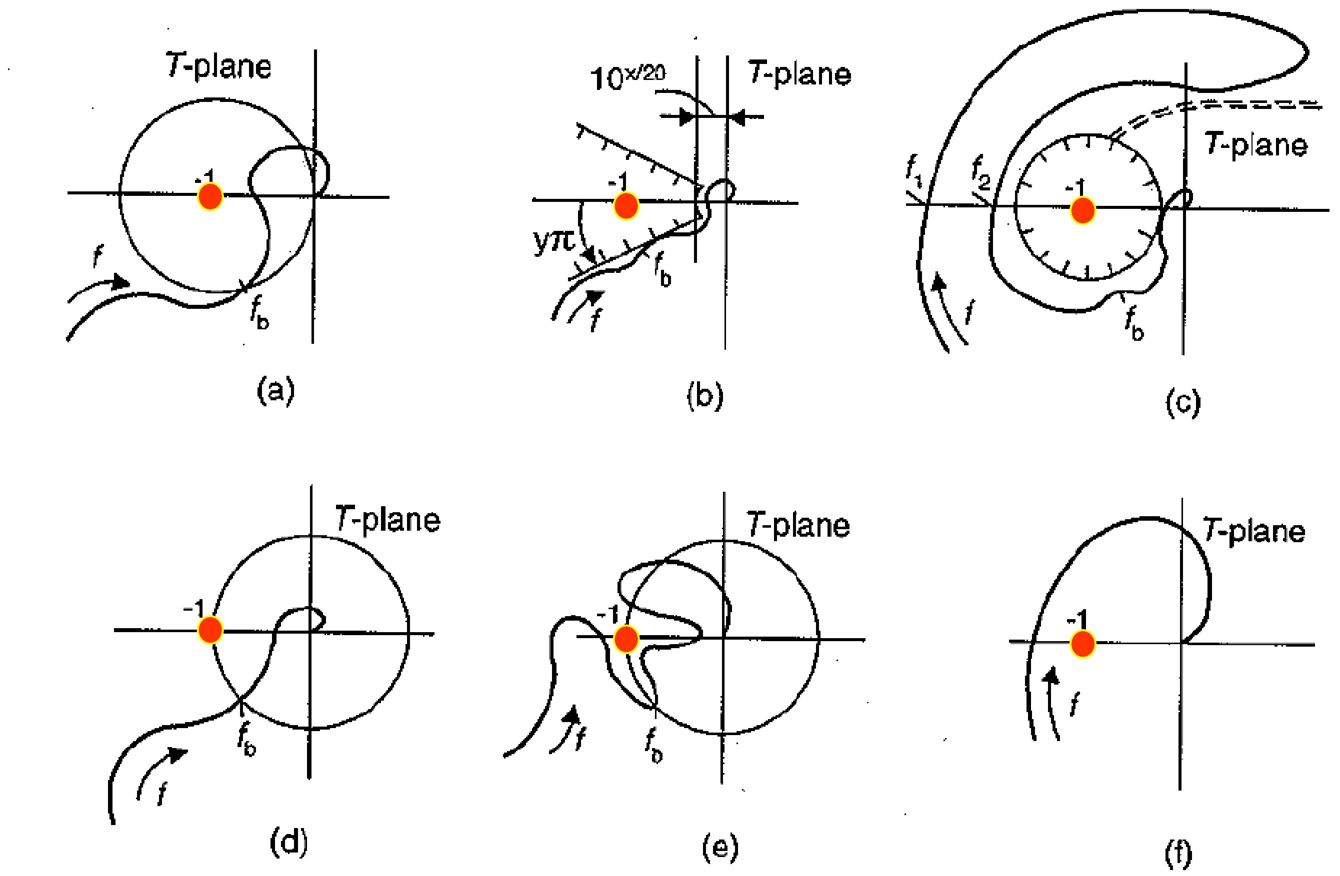
Gain variation v.s. frequency

Phase variation v.s. frequency



Nyquist Diagram: Stability analysis

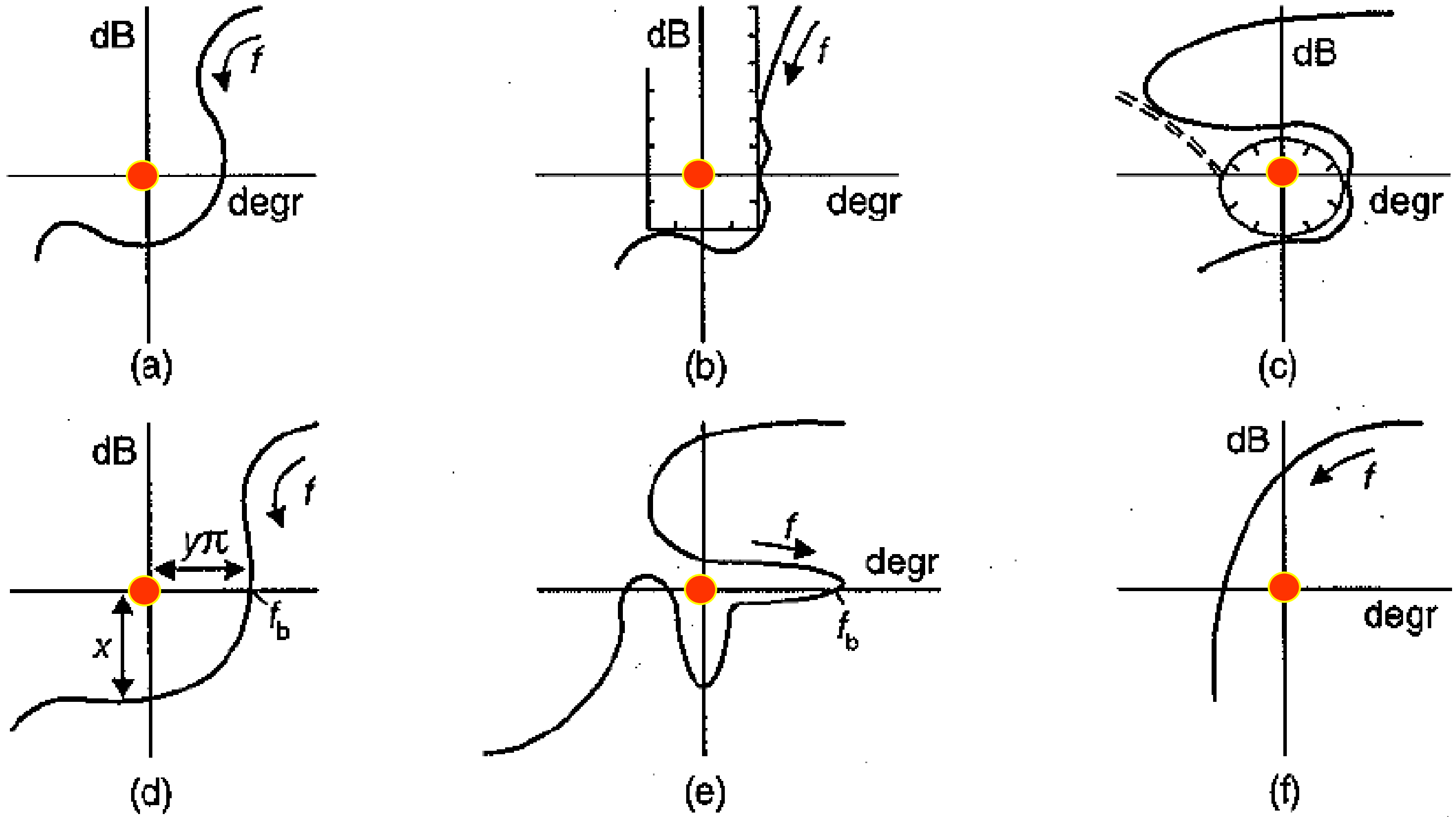
Complex plane : Plot of *GH(jw)* in the complex plane



Critical point (-1, 0) ➔ Gain = 1 et Phase = -180°

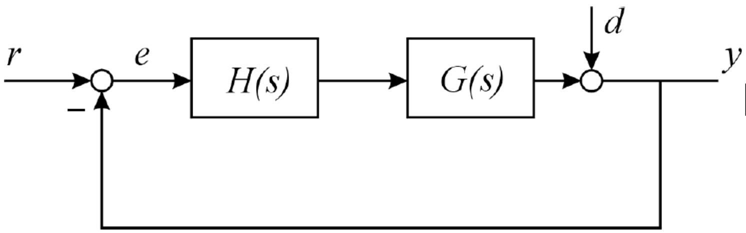
Nichols Diagram: Stability analysis

Plot of gain variation in decibel in function of the phase



Critical point (-180°, 0db) ➔ Gain = 1 et Phase = -180°

1. Disturbance rejection:



$$\frac{y}{d} = \frac{1}{1+GH}$$

Controler effect

- Large effect if GH>>1
- No effect if GH<<1
- Unstability if GH=-1 !!

2. Design trade-off :

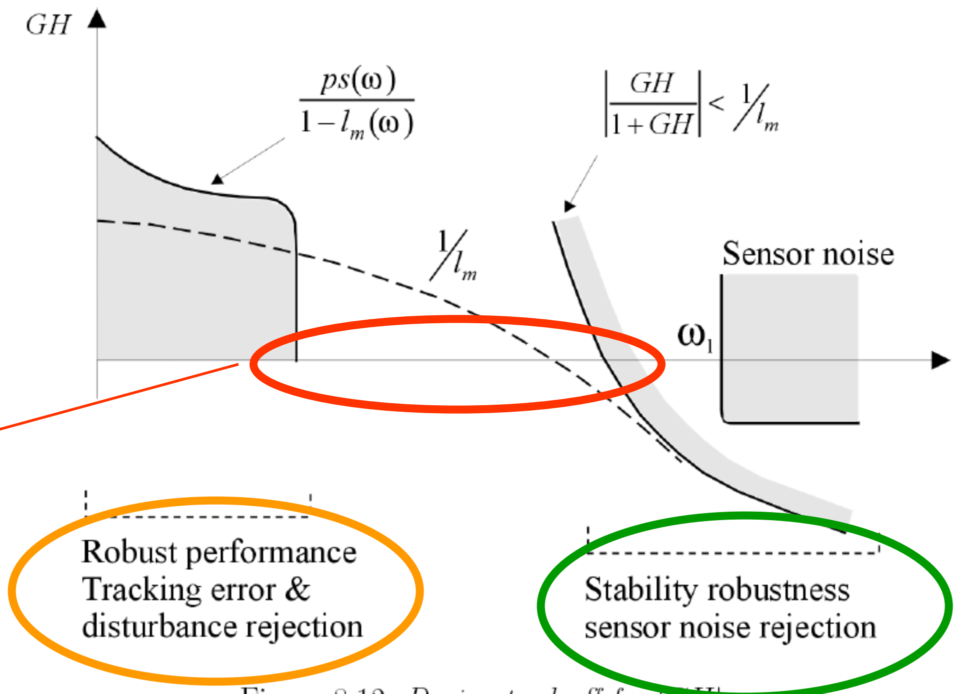


Figure 8.12: *Design tradeoff for |GH|.*

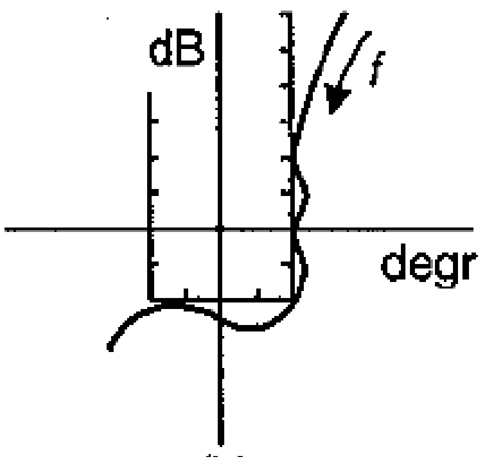
Dangerous zone !

3. 1st Bode integral:

$$\phi(\omega_0) \simeq \frac{\pi}{2}\frac{d\ln|G|}{du}]_{u=0} \Rightarrow \begin{array}{l} -20dB/decade \Rightarrow \phi = -\pi/2 \\ -40dB/decade \Rightarrow \phi = -\pi \end{array}$$

Valid for slow variation of G…

4. Ideal design of Bode :



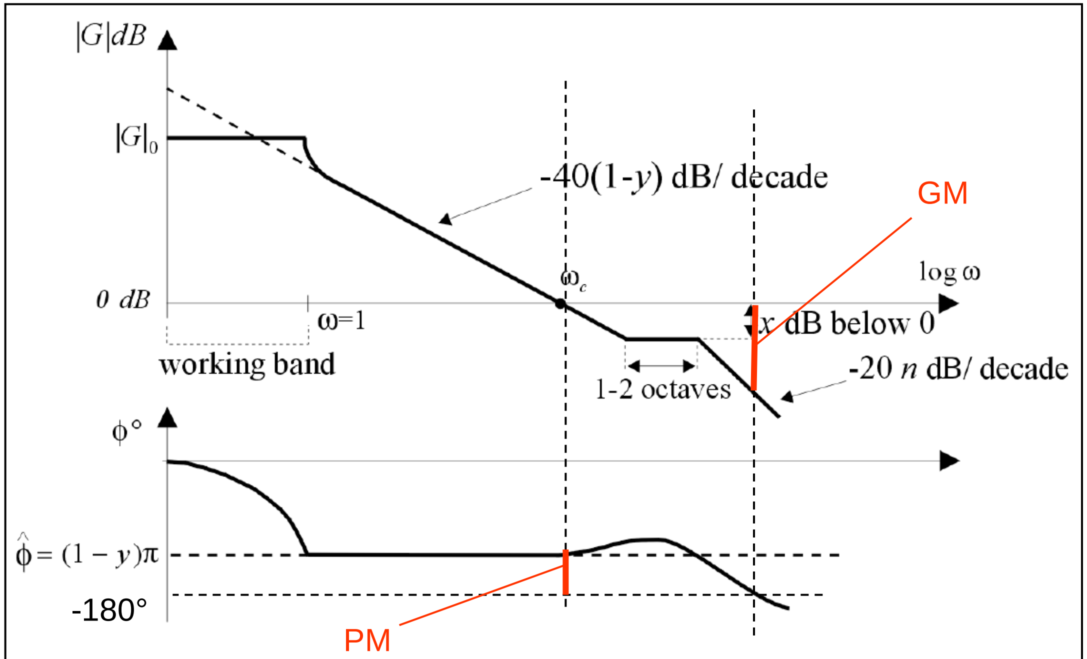
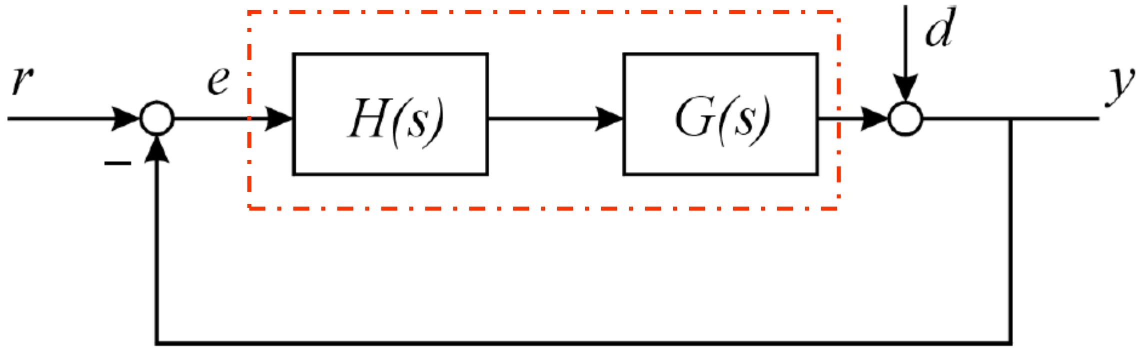


Figure 8.17: *The Bode Ideal Cutoff.*

Active control « classic »

P.8.10 (a) The error specification of the radial positioning system of a CD is 0.2 microns. If the system is subjected to a random disturbance of 200 microns around 25 Hz, compute an estimation of the bandwidth of the control system in order to achieve appropriate disturbance rejection with a reasonable phase margin.

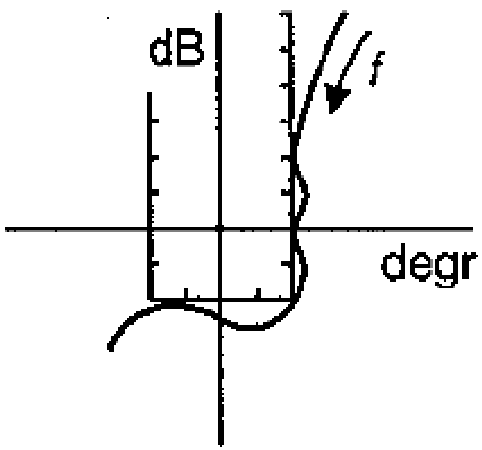


3. 1st Bode integral:

$$\phi(\omega_0) \simeq \frac{\pi}{2}\frac{d\ln|G|}{du}]_{u=0} \Rightarrow \begin{cases} -20dB/decade \Rightarrow \phi = -\pi/2 \\ -40dB/decade \Rightarrow \phi = -\pi \end{cases}$$

Valid for slow variation of G…

4. Ideal design of Bode :



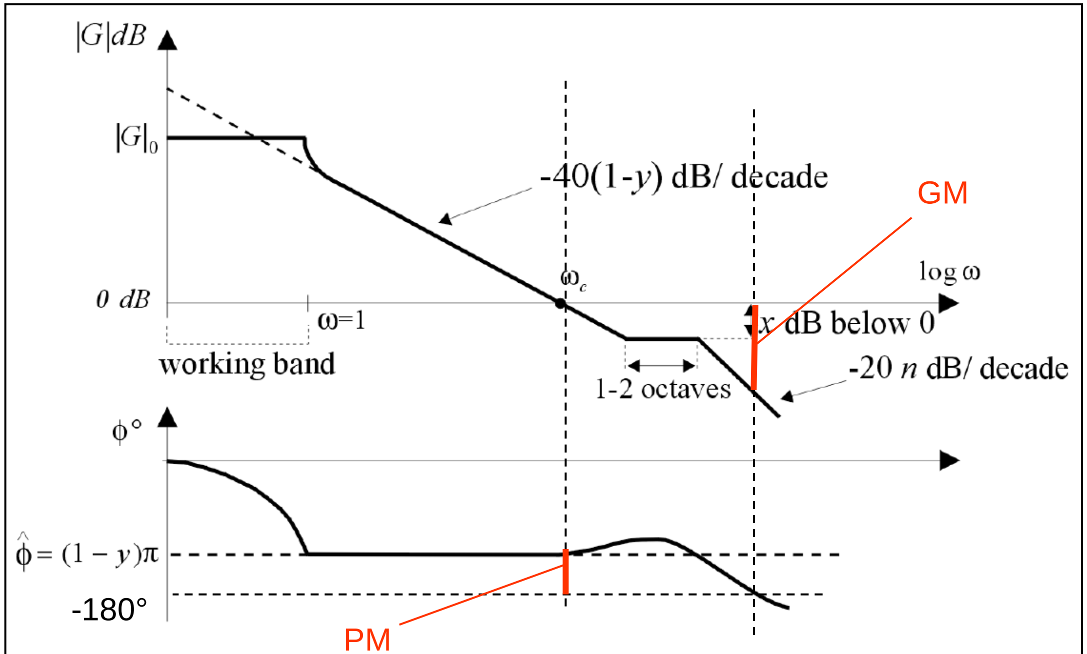


Figure 8.17: *The Bode Ideal Cutoff.*

Solution :

P.8.10 (a) The error specification of the radial positioning system of a CD is 0.2 microns. If the system is subjected to a random disturbance of 200 microns around 25 Hz, compute an estimation of the bandwidth of the control system in order to achieve appropriate disturbance rejection with a reasonable phase margin.

- d = 200 µm
- e_{max} = 0. 2 µm

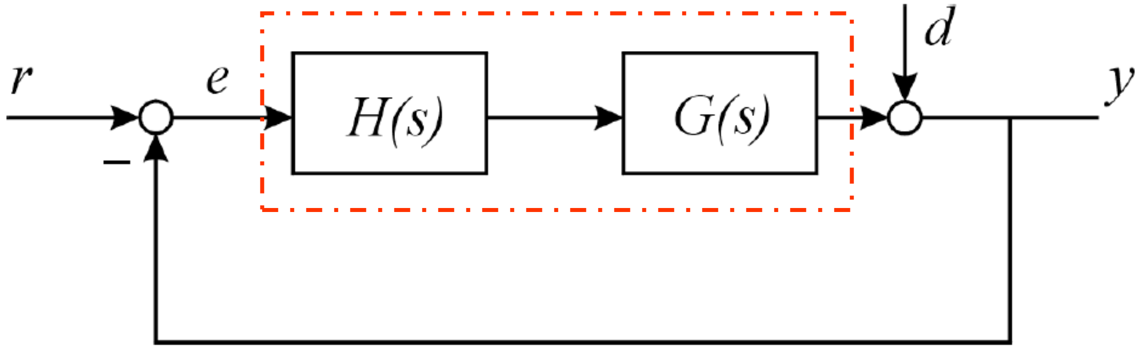
} GH(25 Hz) = 1000 = 60 dB

1. If the slope is -20dB/dec :
   - Φ = -90° → MP = 90°
   - ω_c = 25 Hz + 3 décades = 25 kHz !
2. If we reduce the phase margin :
   - Φ = -120° → MP = 60°
   - ωc = 25 Hz + 2.25 décades = 4.5kHz

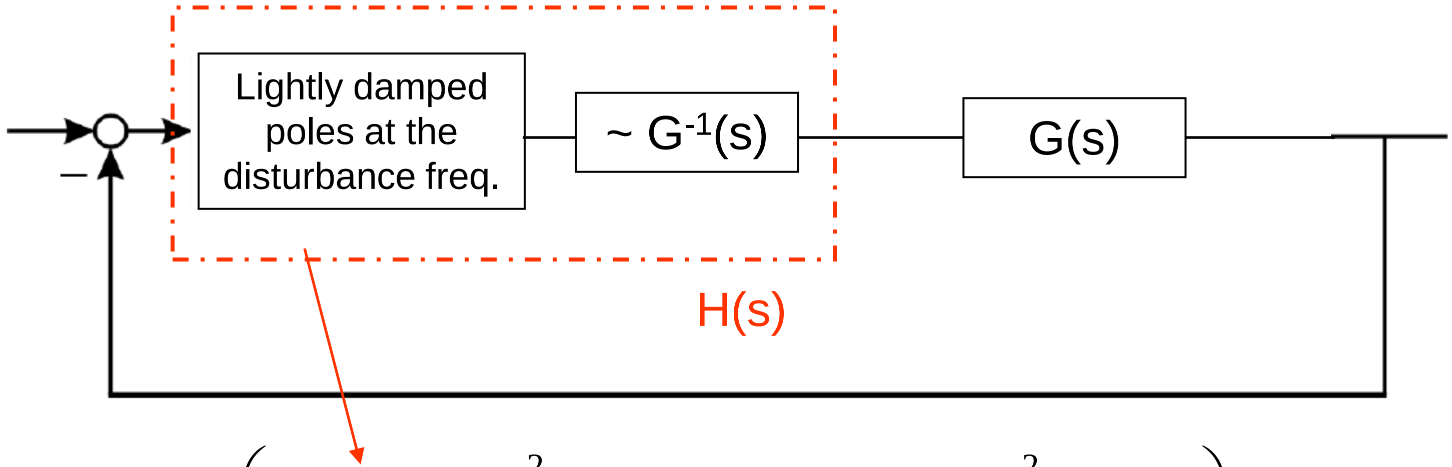
Periodic disturbance

P.8.10 (b) What if the disturbance is known to be periodic? Suppose that the disturbance is the superposition of a perfect sine at 25 Hz and its first harmonic at 50 Hz, and design the appropriate controller with very lightly damped poles (e.g. ξ=0.25%). Next, compute the disturbance rejection if there is a small error in the disturbance frequency. Do the same when the poles of the controller have more damping (e.g. ξ=1%).

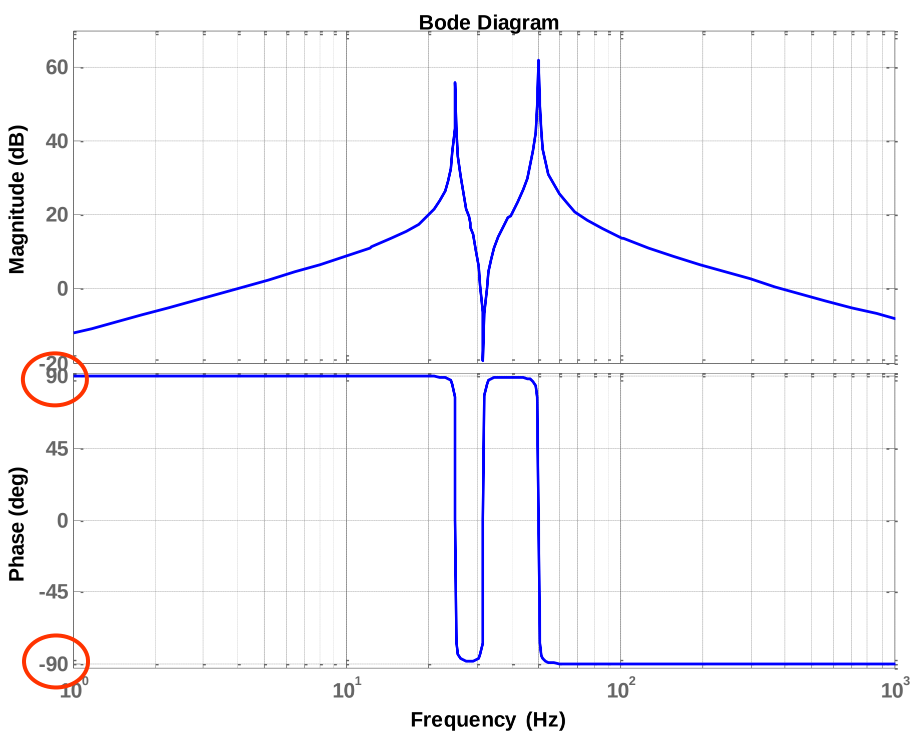
Same idea:



But, we have more information about d and we will take it into consideration.



$$H(s) = s\left(g_1 \frac{\omega_1^2}{s^2 + 2\xi\omega_1 s + \omega_1^2} + g_2 \frac{\omega_2^2}{s^2 + 2\xi\omega_2 s + \omega_2^2} \right)$$



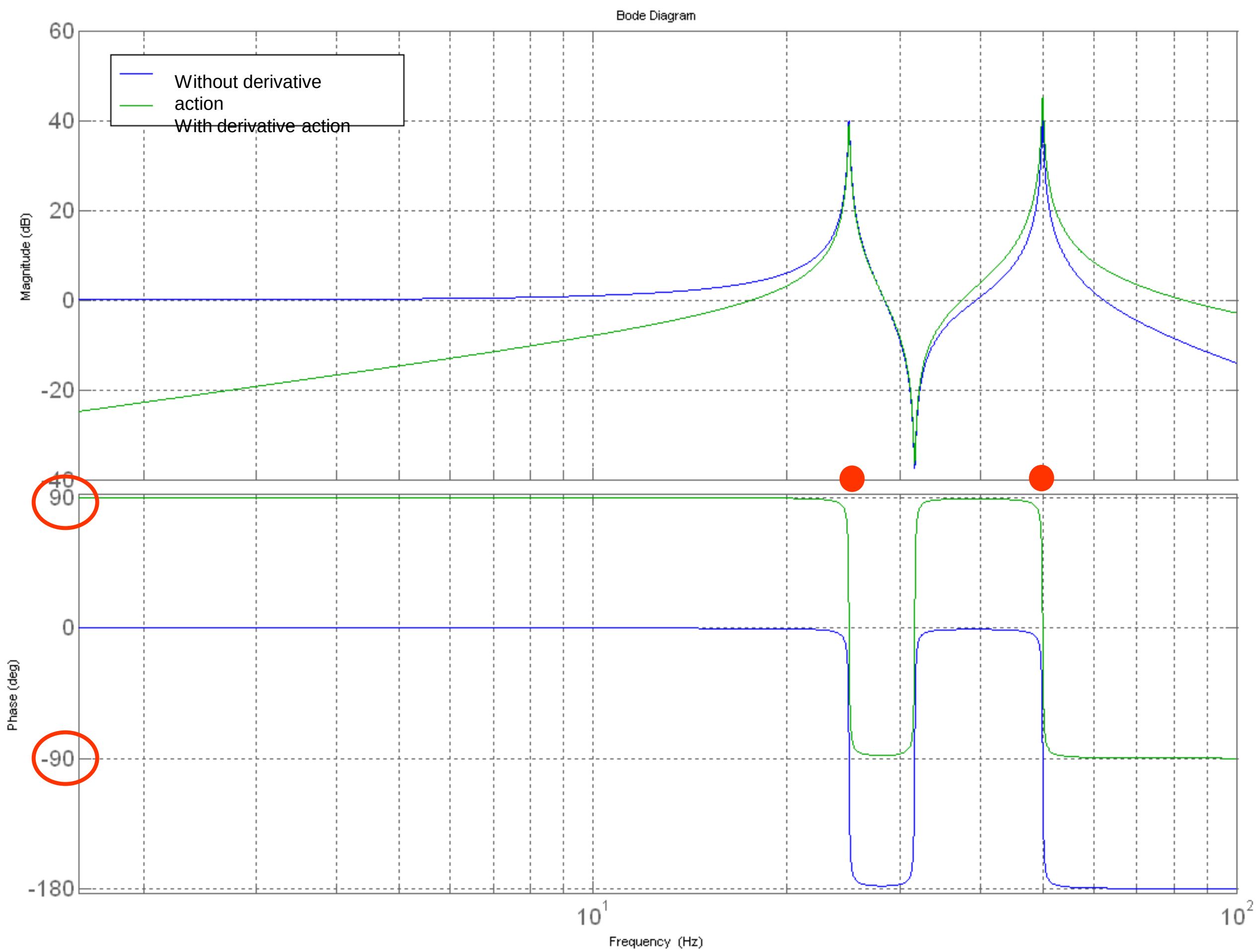
Stable if the model of G(s) is accurate at ± 90° arround the disturbance frequency,

Gain stabilization (i.e. the phase of G(s)H(s) does never exceed ±180°)

Controler synthesis : Open loop GH

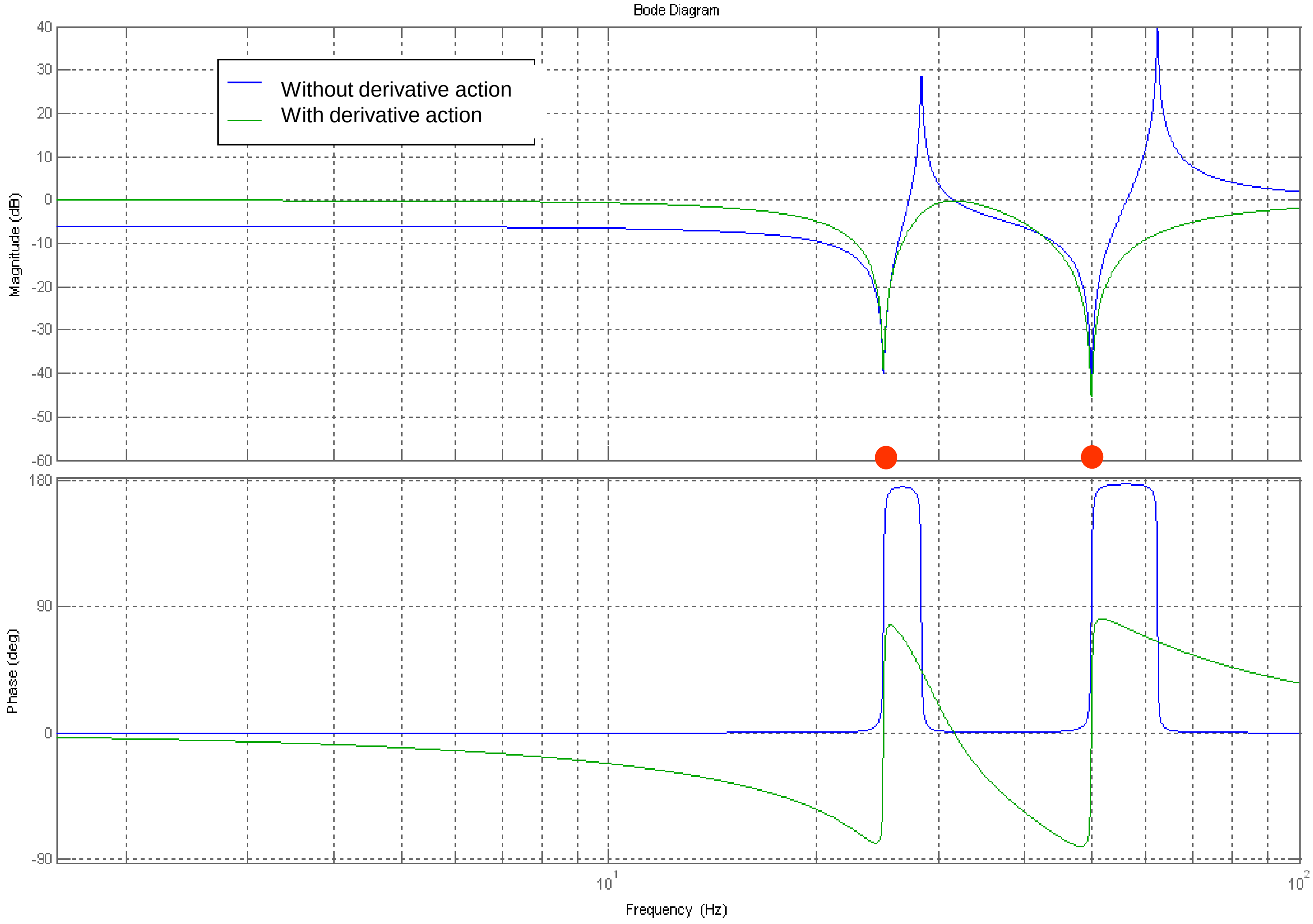
$$H(s)=\left(g_1\frac{\omega_1^2}{s^2+2\xi\omega_1 s+\omega_1^2}+g_2\frac{\omega_2^2}{s^2+2\xi\omega_2 s+\omega_2^2}\right)$$

$$H(s)=s\left(g_1\frac{\omega_1^2}{s^2+2\xi\omega_1 s+\omega_1^2}+g_2\frac{\omega_2^2}{s^2+2\xi\omega_2 s+\omega_2^2}\right)$$

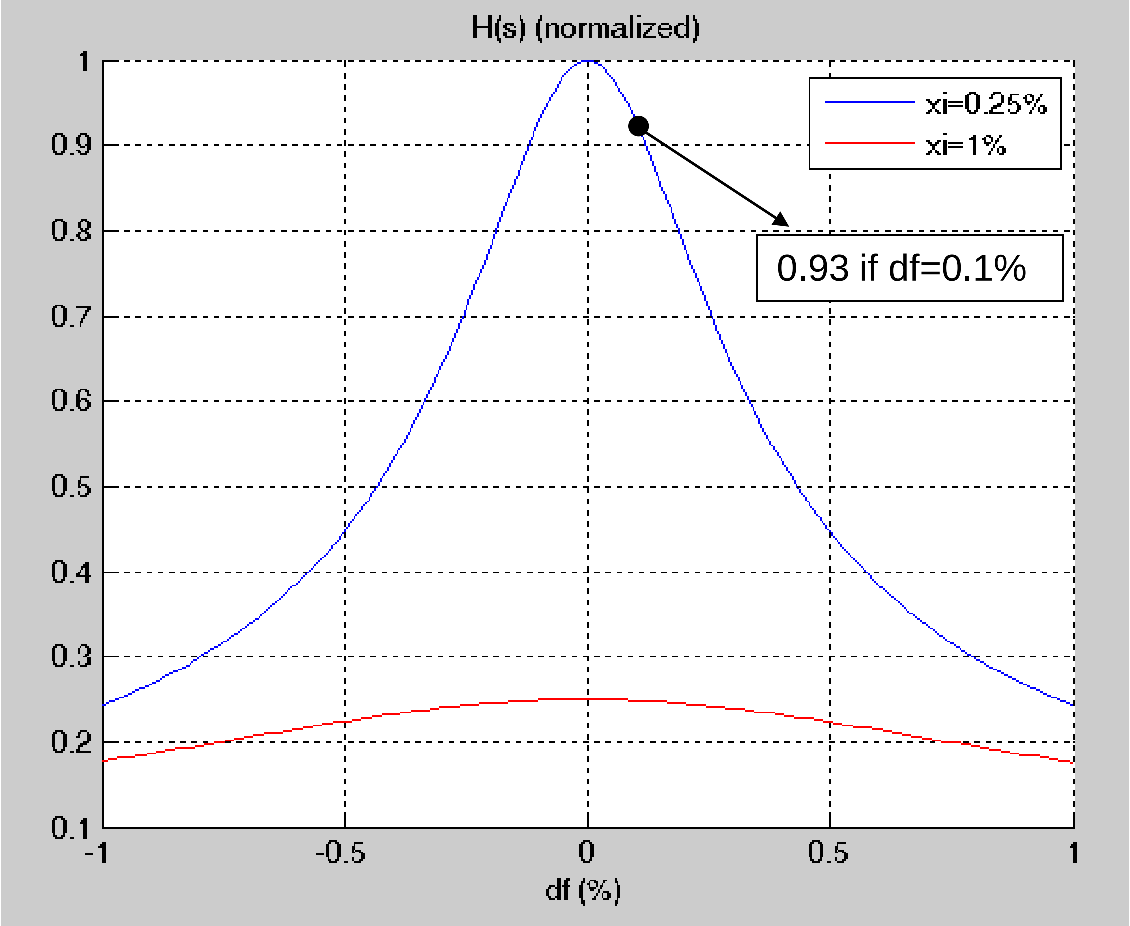


Sensitivity function:

$$S(s) = \frac{y}{d} = \frac{1}{1 + G(s)H(s)}$$



What If the disturbance frequency is not exactly 25 Hz?



Info : Electrical network frequency (in Belgium…) variation is around 0.1%

P.8.6 *Consider a SISO system with the following performance specifications:*

$$|G| > 30dB \qquad \omega < 1\, rad/s$$

$$|G| < -30dB \qquad \omega > 10\, rad/sec$$

Using the first Bode integral, discuss the feasibility of the design.

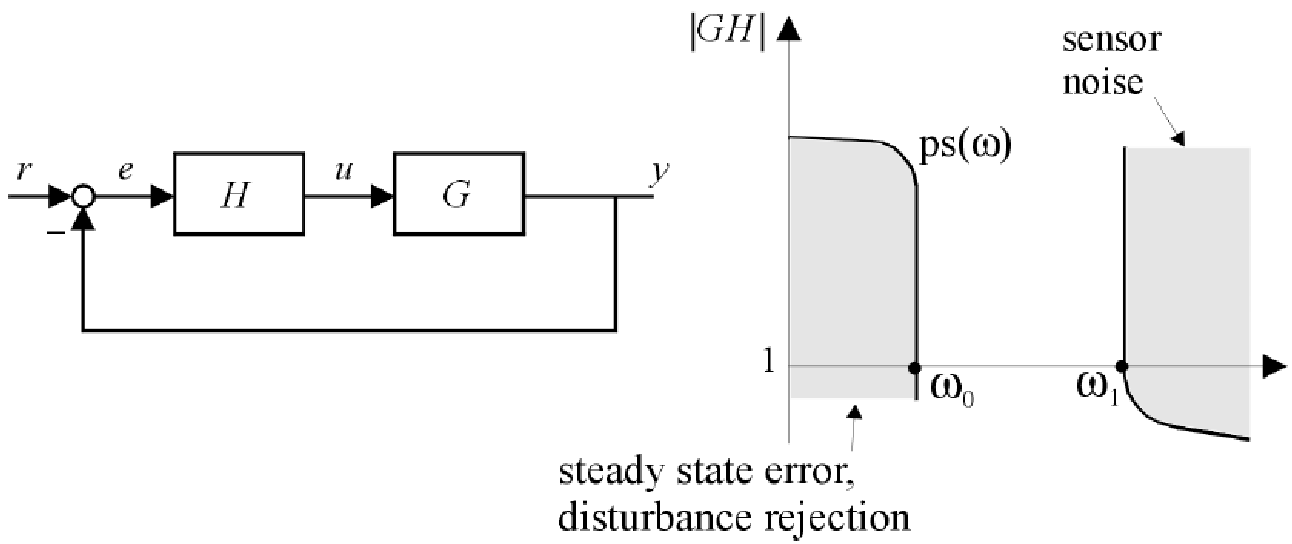
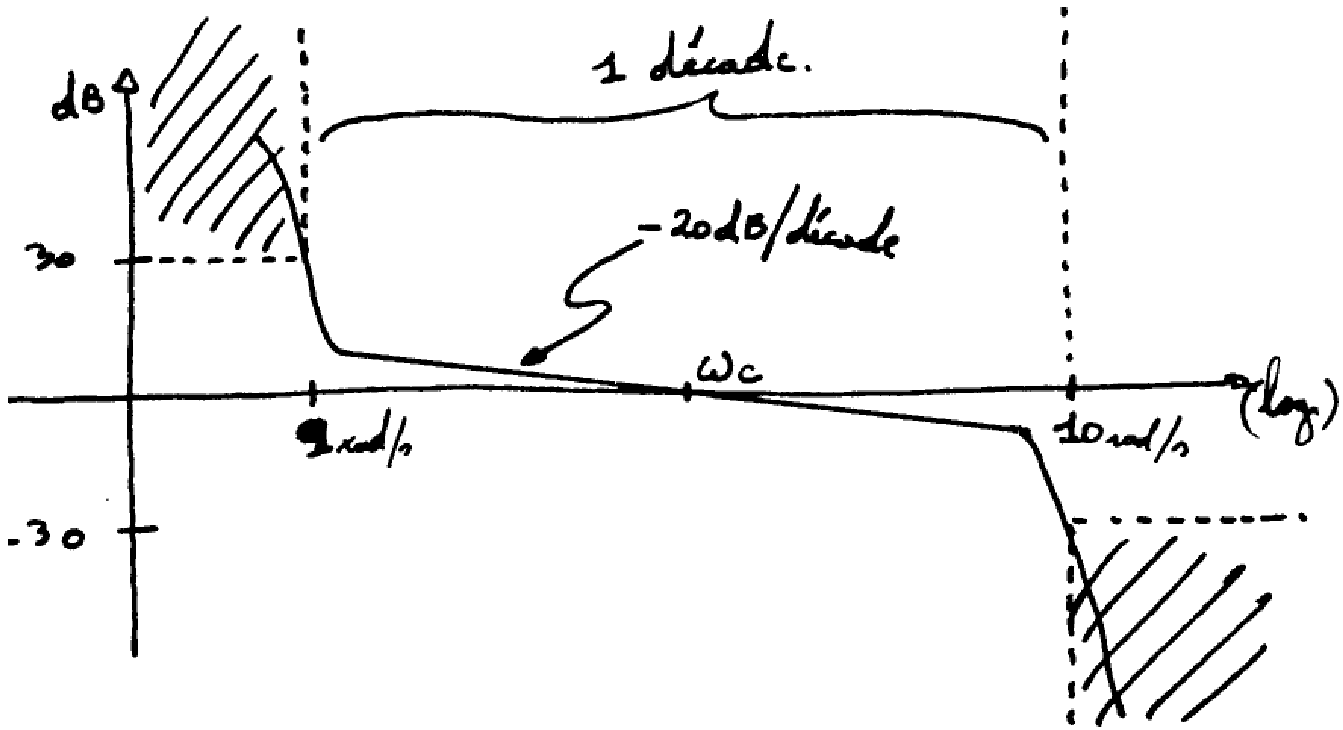


Figure 8.9: *Feedback specification.*

• Impossible to reduce G(s)H(s) of 60 dB in only one decade (PM= -90 ° !)

• One must reduce strongly G(s)H(s) around 1rad/s and 10rad/s, to get an « acceptable » slope at ω_c.



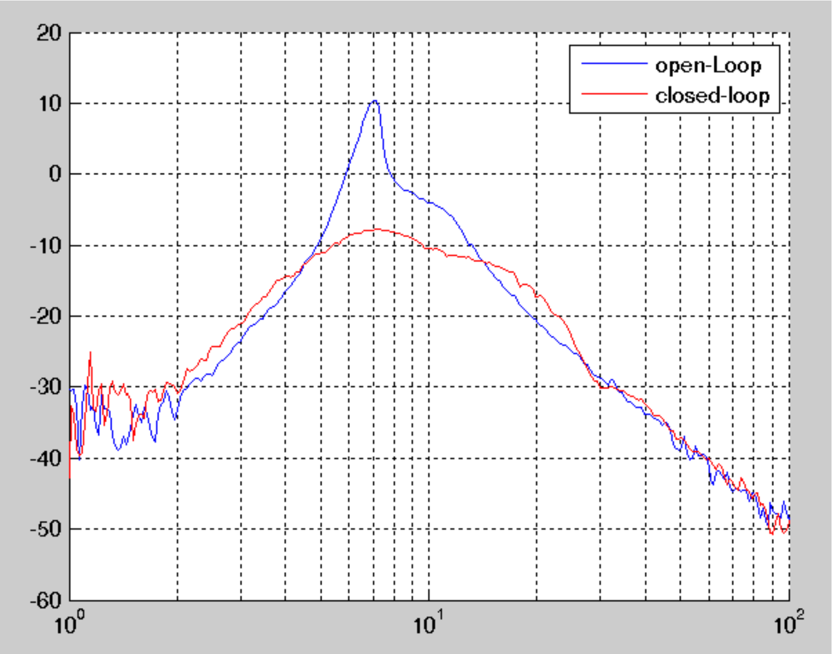
• Possible ? Yes, with high order filters.

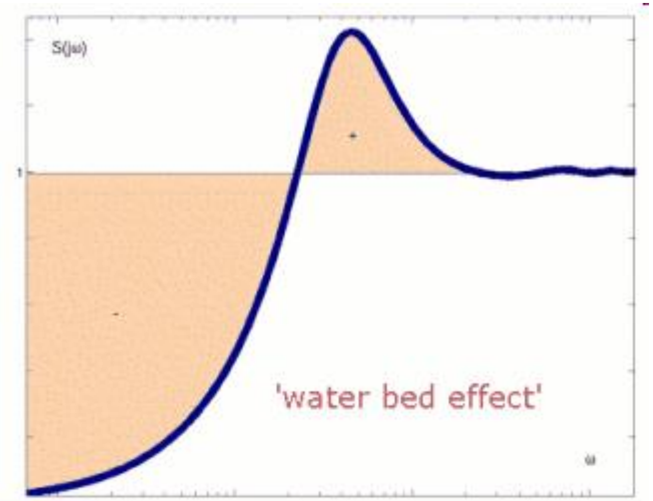
• But difficult and not recommended !

2° Bode integral or « water bed effect»

$$\int_0^\infty \ln|S|\,d\omega = 0 \qquad \text{with:} \qquad S = \frac{1}{1+G(s)H(s)}$$

→ The decrease of S at some frequencies (i.e. disturbance rejection) is always compensated by an increase of S at other frequencies.





There is a direct relationship between the phase margin and the overshoot of the closed loop response,

In general, a phase margin >60° is sufficient to avoid the peak of the closed loop response,

P.8.9 *Consider a stable, minimum phase system $G(s)$ with a high frequency attenuation rate larger than $-20dB/decade$. Using a Nyquist diagram and geometrical arguments, show that there is always a limit frequency above which there is an amplification of the disturbances.*

P.8.9 *Consider a stable, minimum phase system $G(s)$ with a high frequency attenuation rate larger than $-20dB/decade$. Using a Nyquist diagram and geometrical arguments, show that there is always a limit frequency above which there is an amplification of the disturbances.*

Solution :

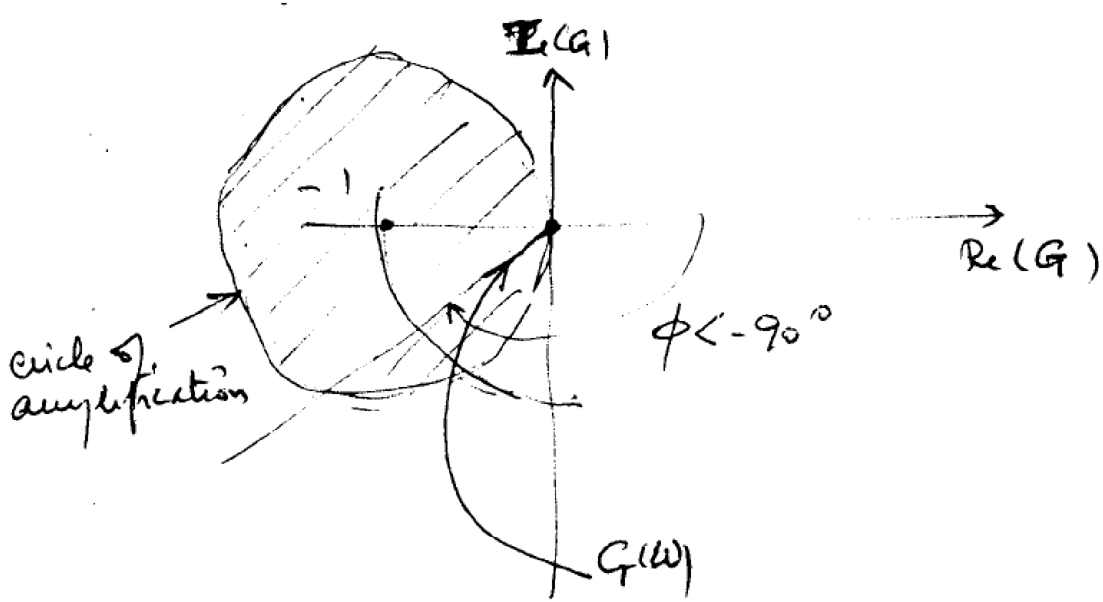
1. $e^* = r - y = \dfrac{1}{1+G}(r-d)$ → amplification if|1+G|<1

2. |1+G| < 1 when G(jω) enters the circle with radius of 1 centered at (-1,0)

3. The slope exceeds -20dB/dec at high frequencies:

   So φ < -90° AND G(jω) → 0 if ω >>>

   So G(jω) crosses the « Amplification circle » at high frequencies.

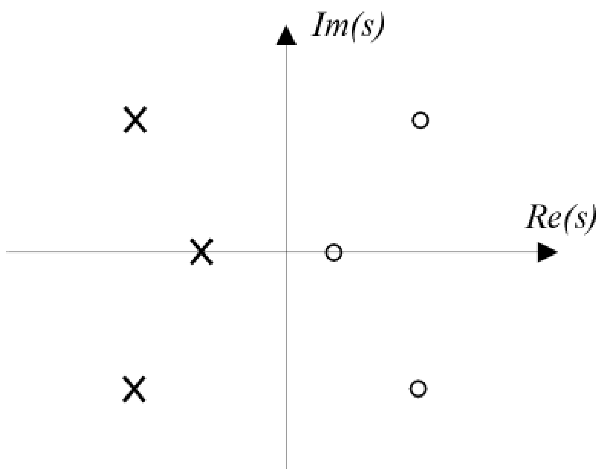


« non-minimum phase » Systems

P.8.7 *Consider a non-minimum phase system with a real zero at* $a = 5\omega_c$. *Following the Bode Ideal Cutoff, draw the magnitude and phase diagrams for* $G_0(s)$ *which produce a gain margin of* x *dB and a phase margin of* $y\pi$ *for* $G(s)$. *Do the same for* $a = 2\omega_c$.

- Non-minimum phase = presence of zeros with positive real part
- These systems cannot be inverted !
- They are treated with « pass-all » filters :

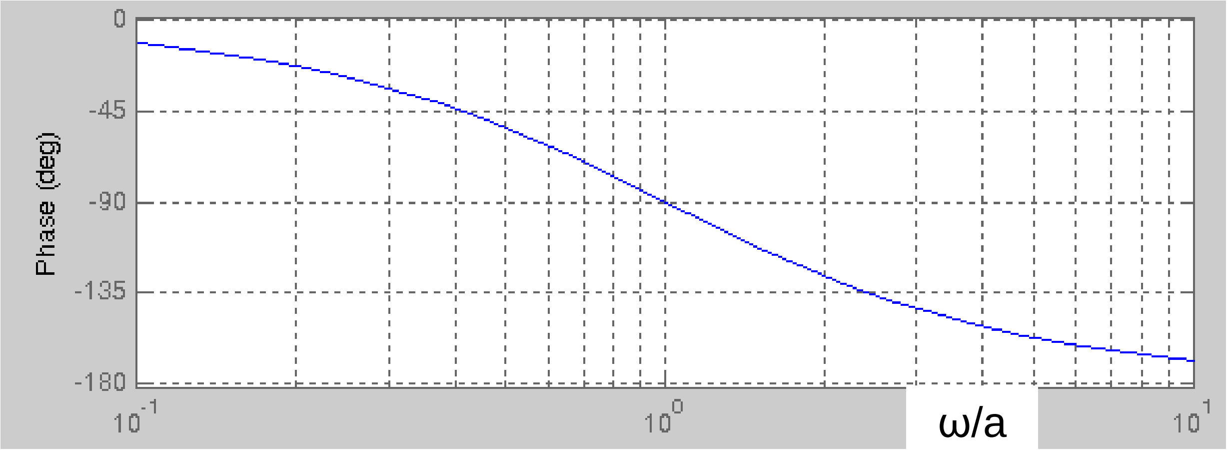
$$G(s) = G_0(s)\,A(s)$$ with: $$A(s) = -\frac{s-a}{s+a}$$



System almost identical to G(s) BUT minimum phase.

Characteristics of A(s) :

1. Magnitude = 1 (« all pass »)
2. Phase :



P.8.7 *Consider a non-minimum phase system with a real zero at $a = 5\omega_c$. Following the Bode Ideal Cutoff, draw the magnitude and phase diagrams for $G_0(s)$ which produce a gain margin of x dB and a phase margin of $y\pi$ for $G(s)$. Do the same for $a = 2\omega_c$.*

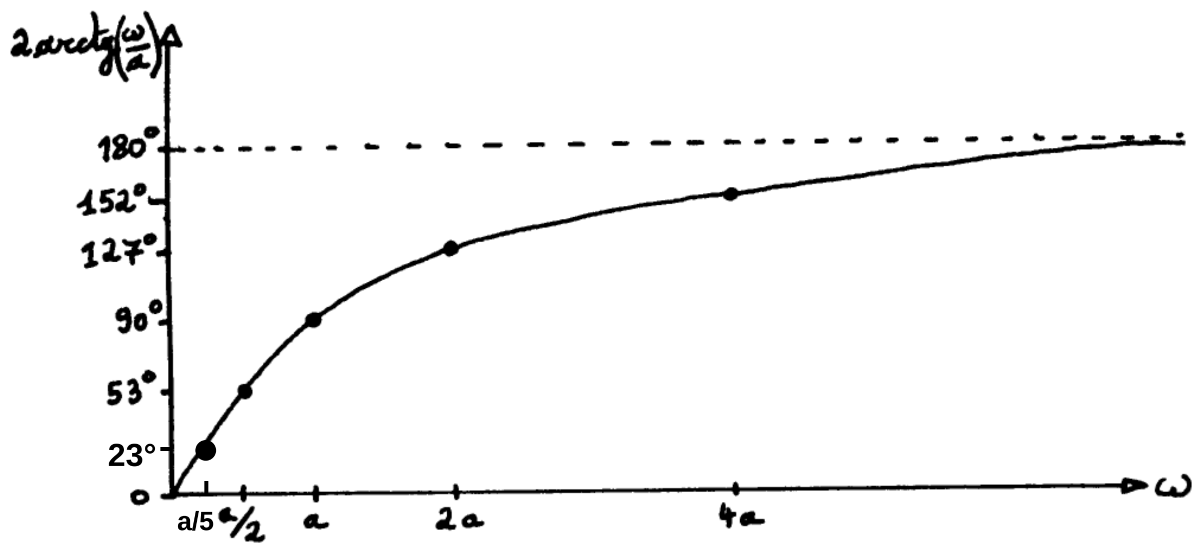
Solution :

a) $|A(\omega)| = 1 \Rightarrow$ "$G(s)$ and $G_o(s)$ have the same magnitude $\forall\omega$."

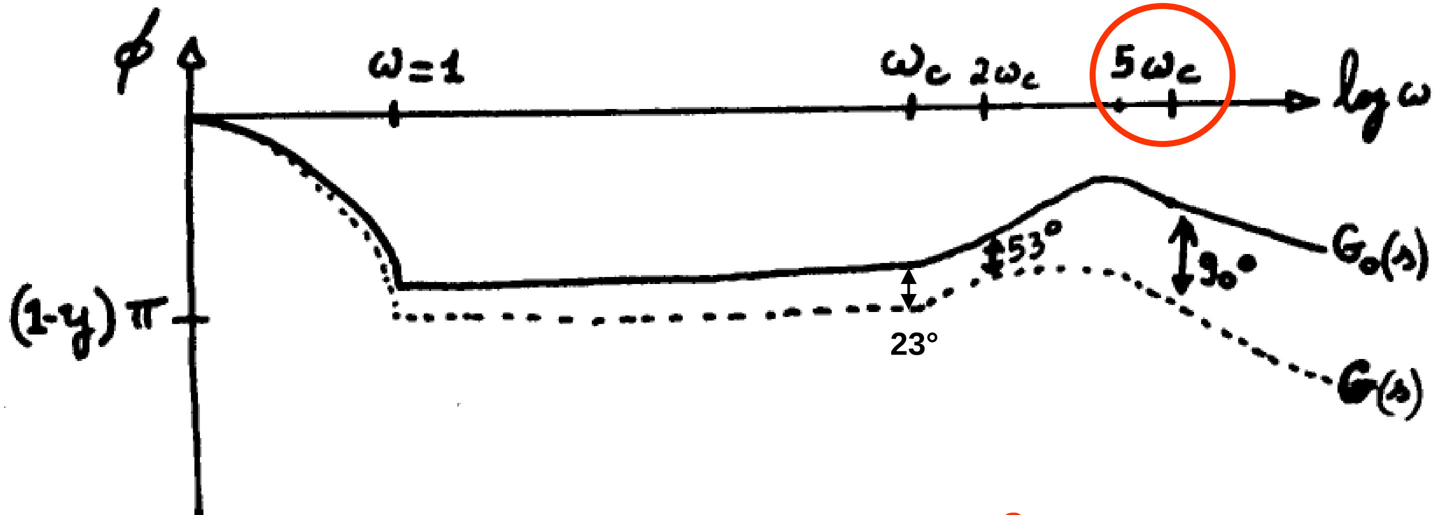
b) $\phi[A(\omega)] = \text{arctg}\left(\frac{-\omega}{a}\right) - \text{arctg}\left(\frac{\omega}{a}\right) = -2\,\text{arctg}\frac{\omega}{a}$

$$\Rightarrow \phi[G_o(s)] = \phi[G(s)] - \phi[A(s)]$$

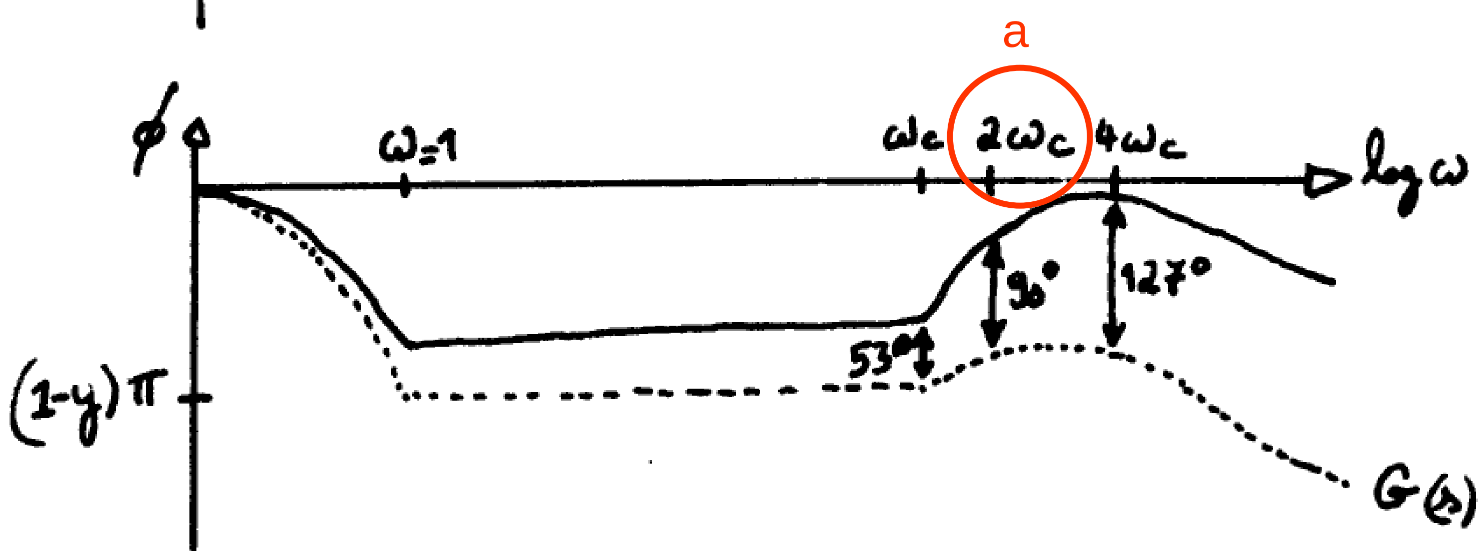
$$= \phi[G(\omega)] + 2\,\text{arctg}\frac{\omega}{a}$$



Solution:



φ
ω=1
ωc
2ωc
5ωc
log ω
53°
90°
23°
(1-y)π
G0(s)
G(s)



a
φ
ω=1
ωc
2ωc
4ωc
log ω
90°
127°
53°
(1-y)π
G(s)