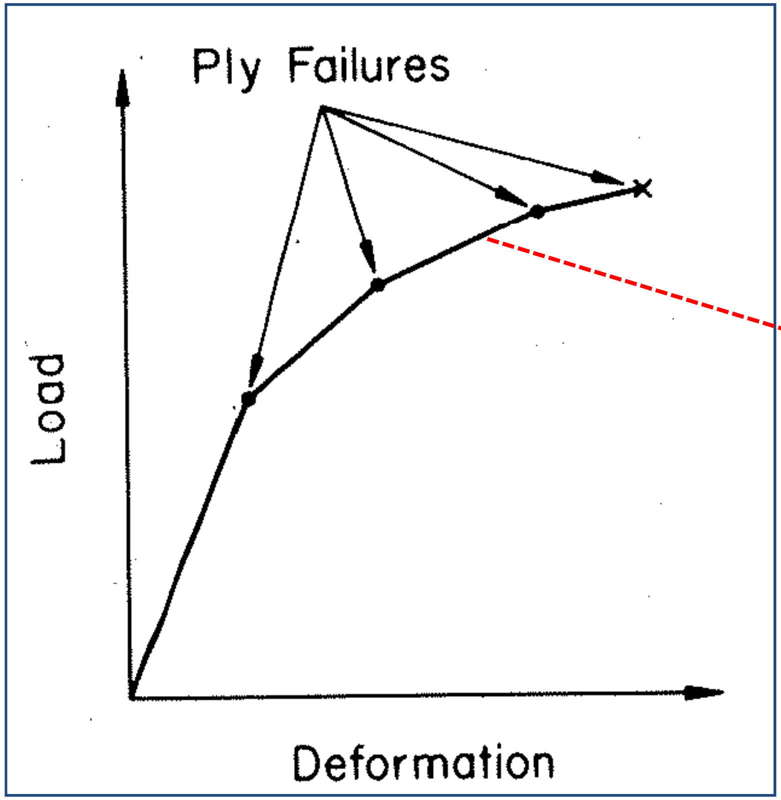
Analysis of laminates after initial failure



$$\begin{Bmatrix} N \\ \hline M \end{Bmatrix} = \left[\begin{array}{c|c} A & B \\ \hline B & D \end{array}\right] \begin{Bmatrix} \epsilon^0 \\ \hline k \end{Bmatrix}$$

$$\begin{Bmatrix} \Delta N \\ \hline \Delta M \end{Bmatrix}_i = \left[\begin{array}{c|c} \bar{A} & \bar{B} \\ \hline \bar{B} & \bar{D} \end{array}\right]_i \begin{Bmatrix} \Delta\epsilon^0 \\ \hline \Delta k \end{Bmatrix}_i$$

Accounting for the modification of stiffness in the failed layers

Common practice: set all lamina properties to zero when failure occurs (gives a conservative estimate of the load carrying capacity)

$$[\bar{Q}]_k = 0$$ **for the failed ply**

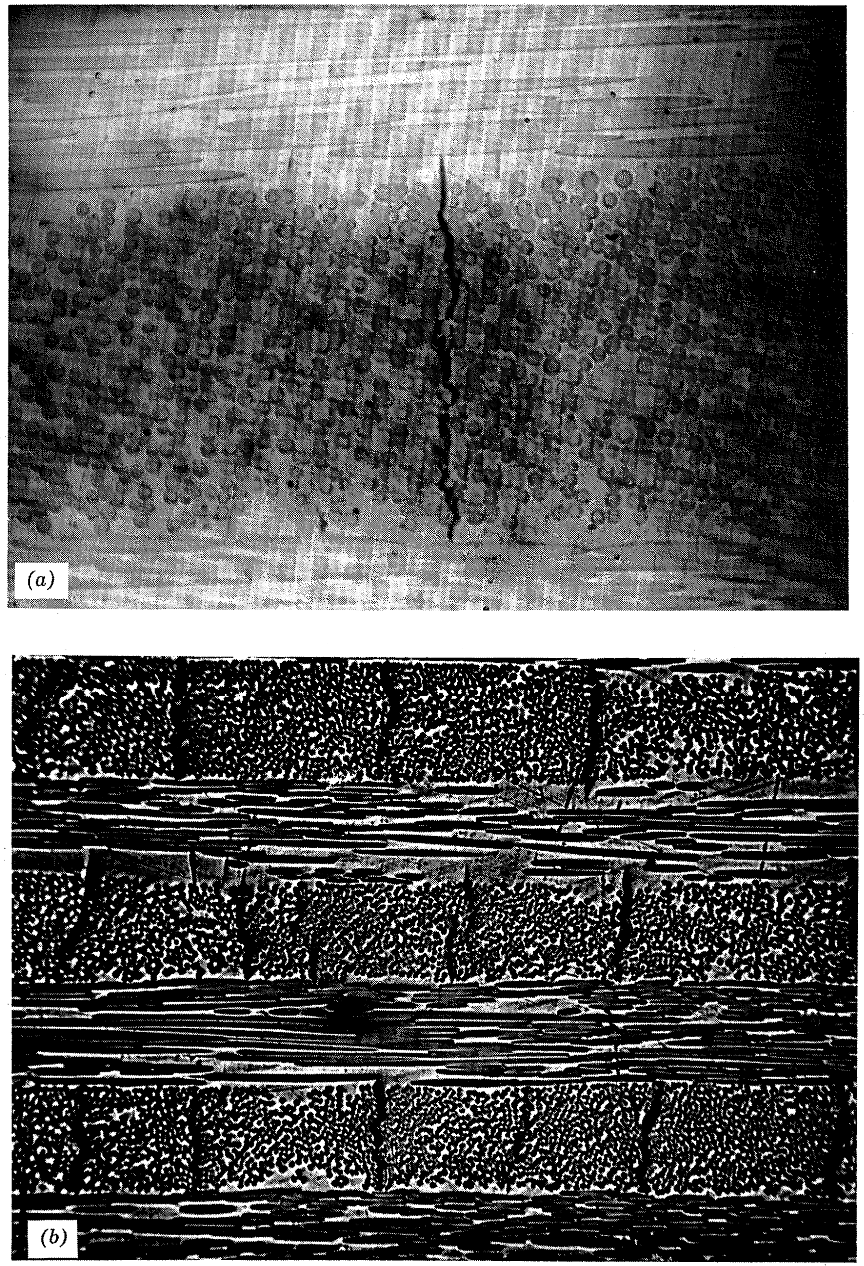


Figure 8.1. Fatigue failure initiation: (*a*) single cross-ply crack and (*b*) multiple crack formation in cross plies.

Example: 5 mm symmetric **cross-ply** constructed from 15 identical laminae
9 plies at 0° and 6 plies at 90°, load in traction N_x

$$Q = \begin{bmatrix} 56 & 4.6 & 0 \\ 4.6 & 18.7 & 0 \\ 0 & 0 & 8.9 \end{bmatrix} \text{GPa}$$

$$\sigma_{LU} = 1050 \text{ MPa}$$
$$\sigma_{TU} = 28 \text{ MPa}$$
$$\tau_{LTU} = 42 \text{ MPa}$$

Step 1: Failure of the 90° plies

$$\text{Thickness of } 0^\circ \text{ plies (9 in number)} = \tfrac{5}{15} \times 9 = 3 \text{ mm}$$
$$\text{Thickness of } 90^\circ \text{ plies (6 in number)} = \tfrac{5}{15} \times 6 = 2 \text{ mm}$$

$$[\bar{Q}]_{0^\circ} = \begin{bmatrix} 56.0 & 4.6 & 0 \\ 4.6 & 18.7 & 0 \\ 0 & 0 & 8.9 \end{bmatrix} \text{GPa}$$

$$[\bar{Q}]_{90^\circ} = \begin{bmatrix} 18.7 & 4.6 & 0 \\ 4.6 & 56.0 & 0 \\ 0 & 0 & 8.9 \end{bmatrix} \text{GPa}$$

$$A_{ij} = 3(\bar{Q}_{ij})_{0^\circ} + 2(\bar{Q}_{ij})_{90^\circ}$$

$$[A] = \begin{bmatrix} 205.4 & 23 & 0 \\ 23 & 168.1 & 0 \\ 0 & 0 & 44.5 \end{bmatrix} \text{GPa} \cdot \text{mm}$$

Extensional stiffness matrix before failure of the 90° ply

Step 2: failure of the first ply (maximum strain theory)
Calculation of the failure strains

Failure stresses

$$\sigma_{LU} = 1050 \text{ MPa}$$

$$\sigma_{TU} = 28 \text{ MPa}$$

$$\tau_{LTU} = 42 \text{ MPa}$$

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \end{Bmatrix}$$

$$Q = \begin{bmatrix} 56 & 4.6 & 0 \\ 4.6 & 18.7 & 0 \\ 0 & 0 & 8.9 \end{bmatrix} \text{GPa}$$

$$Q_{11} = \frac{E_L}{1 - \nu_{LT}\nu_{TL}}$$

$$Q_{22} = \frac{E_T}{1 - \nu_{LT}\nu_{TL}}$$

$$Q_{12} = \frac{\nu_{LT}E_T}{1 - \nu_{LT}\nu_{TL}} = \frac{\nu_{TL}E_L}{1 - \nu_{LT}\nu_{TL}}$$

$$Q_{66} = G_{LT}$$

$$E_L = 54.87 \text{ GPa}$$

$$E_T = 18.32 \text{ GPa}$$

Failure strains

$$\varepsilon_{LU} = \frac{1050 \times 10^{-3}}{54.87} = 0.01914$$

$$\varepsilon_{TU} = \frac{28 \times 10^{-3}}{18.32} = 0.00153$$

The 90° ply will fail when ε_x=0.00153

The load N_x leading to the failure of the 90° ply is solution of

$$\begin{Bmatrix} N_x \\ 0 \\ 0 \end{Bmatrix} = \begin{bmatrix} 205.4 & 23 & 0 \\ 23 & 168.1 & 0 \\ 0 & 0 & 44.5 \end{bmatrix} \begin{Bmatrix} 0.00153 \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

⇨ $N_x = 0.3094 \text{ GPa} \cdot \text{mm} = 309.4 \text{ MPa} \cdot \text{mm}$

Step 3: Behaviour after failure of the first ply

After the failure of the 90° ply, $A_{ij} = 3(\bar{Q}_{ij})_{0°} + 2(\bar{Q}_{ij})_{90°}$

$Q_{90°} = 0$

$$[A] = \begin{bmatrix} 168 & 13.8 & 0 \\ 13.8 & 56.1 & 0 \\ 0 & 0 & 26.7 \end{bmatrix} \text{GPa} \cdot \text{mm}$$

The failure occurs for $\varepsilon_x = 0.01914$ ⇨ $\Delta\varepsilon_x = 0.01914 - 0.00153 = 0.01761$

$$\begin{Bmatrix} \Delta N_x \\ 0 \\ 0 \end{Bmatrix} = \begin{bmatrix} 168 & 13.8 & 0 \\ 13.8 & 56.1 & 0 \\ 0 & 0 & 26.7 \end{bmatrix} \begin{Bmatrix} 0.01761 \\ \Delta\varepsilon_y \\ \Delta\gamma_{xy} \end{Bmatrix}$$

$\Delta N_x = 2898.7 \text{ MPa} \cdot \text{mm}$

$N_x = 2898.7 + 309.4 = 3208.1 \text{ MPa} \cdot \text{mm}$

Flowchart for laminate strength analysis