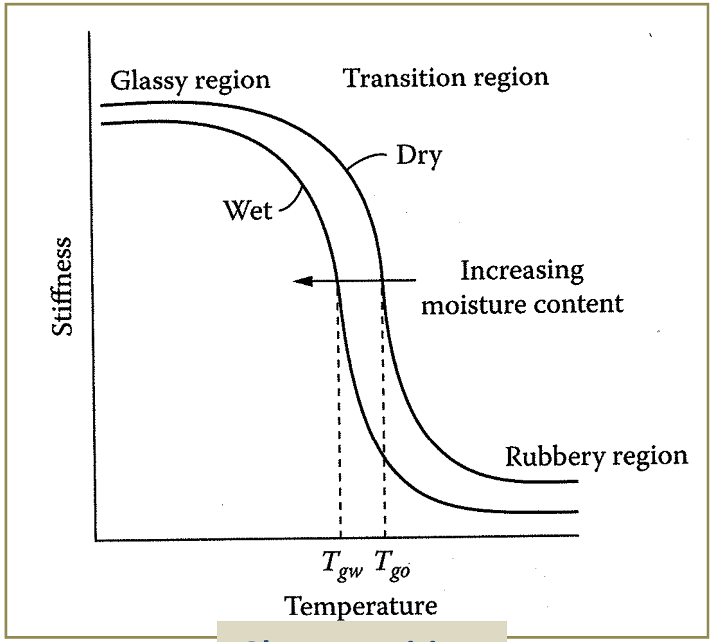
Hygrothermal stresses in laminates

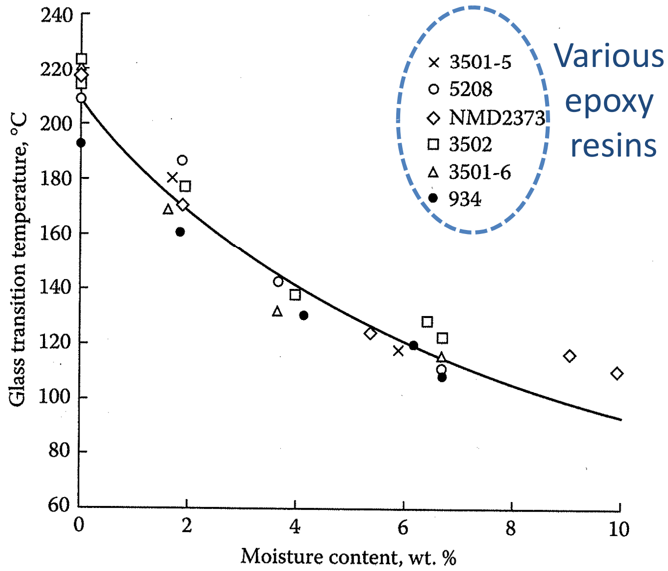
•Changing environment conditions (temperature and moisture) have an important effect on the properties which are matrix dominated.

•Change in temperature and moisture content induces swelling of the polymer matrix.

Temperature effect on polymers



Glass transition temperature



The maximum usage temperature is slightly smaller than T_g

Table 3.1 Typical properties of unidirectional-fiber-reinforced epoxy resins

Property	E-Glass	Kevlar 49	Graphite (Thornel 300)
	Fiber type		
Fiber volume fraction	46	60–65	63
Specific gravity	1.80	1.38	1.61
Tensile strength, 0° (MPa)	1104	1310	1725
Tensile modulus, 0° (GPa)	39	83	159
Tensile strength, 90° (MPa)	36	39	42
Tensile modulus, 90° (GPa)	10	5.6	10.9
Compression strength, 0° (MPa)	600	286	1366
Compression modulus, 0° (GPa)	32	73	138
Compression strength, 90° (MPa)	138	138	230
Compression modulus, 90° (GPa)	8	5.6	11
In-plane shear strength (MPa)	—	60	95
In-plane shear modulus (GPa)	—	2.1	6.4
Longitudinal Poisson ratio (ν_{LT})	0.25	0.34	0.38
Interlaminar shear strength (MPa)	31	69	113
Longitudinal coefficient of thermal expansion (10^{-6}/°C)	5.4	−2.3 to −4.0[a]	0.045
Transverse coefficient of thermal expansion (10^{-6}/°C)	36	35[b]	20.2

[a] −79°C to +100°C.
[b] −195°C to +120°C.

- Thermal expansion of unidirectional composites is strongly anisotropic
- Longitudinal CTE of Kevlar has a negative value
- Longitudinal CTE of Graphite composite close to zero

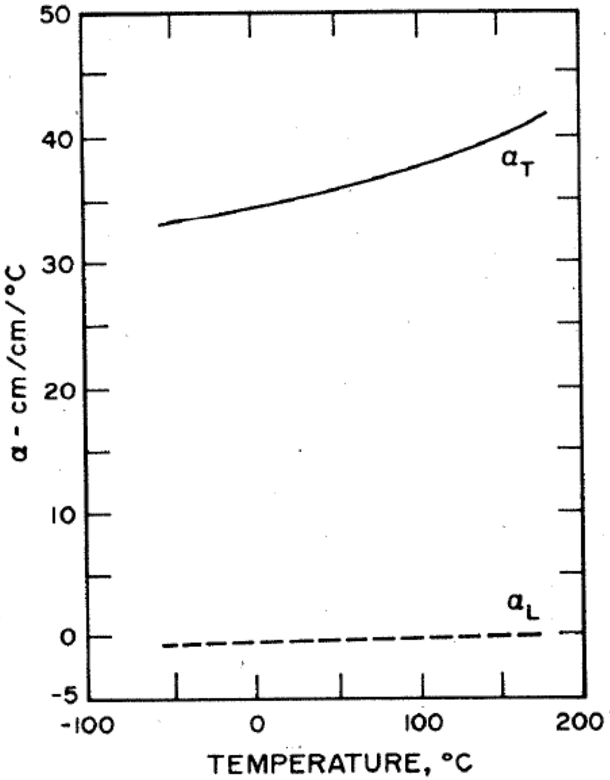


Figure 3.33. Instantaneous coefficient of thermal expansion for unidirectional high-modulus graphite epoxy. Source: Northrop Corporation [49].

Table A.4.1 Properties of some commercial fiber composites

Material description[a]	Fiber volume fraction, V_f	Density (g/cm^3)	Elastic constants E_L (GPa)	E_T (GPa)	ν_{LT}	G_{LT} (GPa)	Strengths σ_{LU} (MPa)	σ'_{LU} (MPa)	σ_{TU} (MPa)	σ'_{TU} (MPa)	τ_{LTU} (MPa)	Hygrothermal expansion coefficients α_L (10^{-6}/°C)	α_T (10^{-6}/°C)	β_L	β_T
Carbon–epoxy T300/N5208	0.70	1.60	181.0	10.30	0.28	7.17	1500	1500	40	246	68	0.02	22.5	0	0.6
Carbon–epoxy AS/H3501	0.66	1.60	138.0	8.96	0.30	7.10	1447	1447	51.7	206	93	−0.3	28.1	0	0.4
Carbon–PEEK AS4/APC2	0.66	1.60	134.0	8.90	0.28	5.10	2130	1100	80	200	160	—	—	—	—
Carbon–epoxy IM6/epoxy	0.66	1.60	203.0	11.20	0.32	8.40	3500	1540	56	150	98	—	—	—	—
Carbon–epoxy T300/Fiberite 934	0.60	1.50	148.0	9.65	0.30	4.55	1314	1220	43	168	48	—	—	—	—
Boron–epoxy B-4/N5505	0.50	2.00	204.0	18.50	0.23	5.59	1260	2500	61	202	67	6.10	30.30	0	0.6
Glass–epoxy E-glass–epoxy	0.45	1.80	38.6	8.27	0.26	4.14	1062	610	31	118	72	8.60	22.10	0	0.6
Aramid–epoxy Kevlar 49–epoxy	0.60	1.46	76.0	5.50	0.34	2.30	1400	235	12	53	34	−4.00	79.0	0	0.6
Carbon–epoxy T300/Fiberite 934 (13-mil)	0.60	1.50	74.0	74.0	0.05	4.55	499	352	458	352	46	—	—	—	—
Carbon–epoxy T-300/Fiberite 934 (7-mil)	0.60	1.50	66.0	66.0	0.04	4.10	375	279	368	278	46	—	—	—	—

[a] The first eight materials are unidirectional, while the last two are fabric reinforced.

$$\varepsilon_L^T = \alpha_L \, \Delta T \qquad \varepsilon_L^H = \beta_L \, \Delta C$$

$$\varepsilon_T^T = \alpha_T \, \Delta T \qquad \varepsilon_T^H = \beta_T \, \Delta C$$

$$C = \frac{\text{Moisture mass}}{\text{Mass of dry material}} \times 100$$

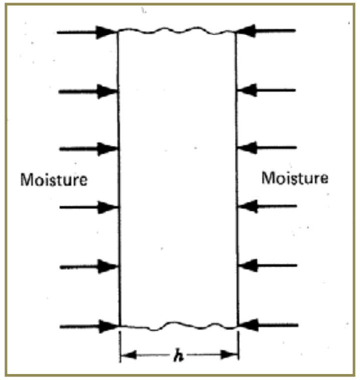
Thermal expansion:
Thermal strains for an orthotropic material in (L,T) frame:
(no thermal shear in L-T frame)

$$\varepsilon_L^T = \alpha_L \, \Delta T$$
$$\varepsilon_T^T = \alpha_T \, \Delta T$$

Hygroscopic expansion:
Changes in moisture concentration are responsible for swelling of the matrix material
Moisture-induced strains in orthotropic material in (L,T) frame:
(no moisture-induced shear in L-T frame)

$$\varepsilon_L^H = \beta_L \, \Delta C$$
$$\varepsilon_T^H = \beta_T \, \Delta C$$

$$C = \frac{\text{Moisture mass}}{\text{Mass of dry material}} \times 100$$



Moisture diffusion is governed by the diffusion equation

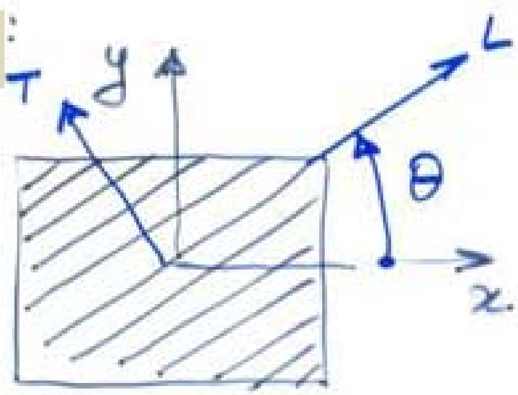
$$\frac{\partial C}{\partial t} = D_z \frac{\partial^2 C}{\partial z^2}$$

Fick's second law
D_z = mass diffusivity along z

Very similar to the heat conduction equation:

$$\varrho C_m \frac{\partial T}{\partial t} = k_z \frac{\partial^2 T}{\partial z^2}$$

Change of coordinates of hygrothermal expansion coefficients



Strain tensor:

$$\begin{Bmatrix} \epsilon_L \\ \epsilon_T \\ \frac{1}{2}\gamma_{LT} \end{Bmatrix} = [T] \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \frac{1}{2}\gamma_{xy} \end{Bmatrix}$$

$$[T] = \begin{bmatrix} \cos^2\theta & \sin^2\theta & 2\sin\theta\cos\theta \\ \sin^2\theta & \cos^2\theta & -2\sin\theta\cos\theta \\ -\sin\theta\cos\theta & \sin\theta\cos\theta & \cos^2\theta - \sin^2\theta \end{bmatrix}$$

$$[T(\theta)]^{-1} = [T(-\theta)]$$

$$\begin{Bmatrix} \alpha_x \\ \alpha_y \\ \frac{1}{2}\alpha_{xy} \end{Bmatrix} = [T]^{-1} \begin{Bmatrix} \alpha_L \\ \alpha_T \\ 0 \end{Bmatrix}$$

Thermal shear

$$\frac{1}{2}\alpha_{xy} = \sin\theta\cos\theta\,(\alpha_L - \alpha_T)$$

$$\begin{Bmatrix} \beta_x \\ \beta_y \\ \frac{1}{2}\beta_{xy} \end{Bmatrix} = [T]^{-1} \begin{Bmatrix} \beta_L \\ \beta_T \\ 0 \end{Bmatrix}$$

Thermal strains

$$\begin{Bmatrix} \varepsilon_x^T \\ \varepsilon_y^T \\ \gamma_{xy}^T \end{Bmatrix} = \begin{Bmatrix} \alpha_x \, \Delta T \\ \alpha_y \, \Delta T \\ \alpha_{xy} \, \Delta T \end{Bmatrix}$$

Hygroscopic strain

$$\begin{Bmatrix} \varepsilon_x^H \\ \varepsilon_y^H \\ \gamma_{xy}^H \end{Bmatrix} = \begin{Bmatrix} \beta_x \, \Delta C \\ \beta_y \, \Delta C \\ \beta_{xy} \, \Delta C \end{Bmatrix}$$

- Do not produce a resultant force or moment when the body is completely free to expand, bend and twist.
- An individual lamina is restrained by the other laminae and is not free to expand. This induces thermal stresses.
- The thermal stresses are internal stresses: they are **self-equilibrated.**

Total strains = mechanical strains + hygrothermal strains

Mechanical strains

$$\begin{Bmatrix} \varepsilon_x^M \\ \varepsilon_y^M \\ \gamma_{xy}^M \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} - \begin{Bmatrix} \varepsilon_x^T \\ \varepsilon_y^T \\ \gamma_{xy}^T \end{Bmatrix} - \begin{Bmatrix} \varepsilon_x^H \\ \varepsilon_y^H \\ \gamma_{xy}^H \end{Bmatrix}$$

Total lamina strains
Follow the kinematics
of Kirchhoff plates:
Linear over the thhickness

$$\begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \epsilon_x^0 \\ \epsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z \begin{Bmatrix} k_x \\ k_y \\ k_{xy} \end{Bmatrix}$$

$$\begin{Bmatrix} \varepsilon_x^M \\ \varepsilon_y^M \\ \gamma_{xy}^M \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 + zk_x \\ \varepsilon_y^0 + zk_y \\ \gamma_{xy}^0 + zk_{xy} \end{Bmatrix} - \begin{Bmatrix} \alpha_x \Delta T \\ \alpha_y \Delta T \\ \alpha_{xy} \Delta T \end{Bmatrix} - \begin{Bmatrix} \beta_x \Delta C \\ \beta_y \Delta C \\ \beta_{xy} \Delta C \end{Bmatrix}$$

Mechanical strains (associated with stresses)

$$\{\varepsilon^M\} = \{\varepsilon^0\} + \{k\}z - \{\alpha\}\Delta T - \{\beta\}\Delta C$$

$$\{\sigma\} = [\bar{Q}]\{\varepsilon^M\} = [\bar{Q}](\{\varepsilon^0\} + \{k\}z - \{\alpha\}\Delta T - \{\beta\}\Delta C)$$

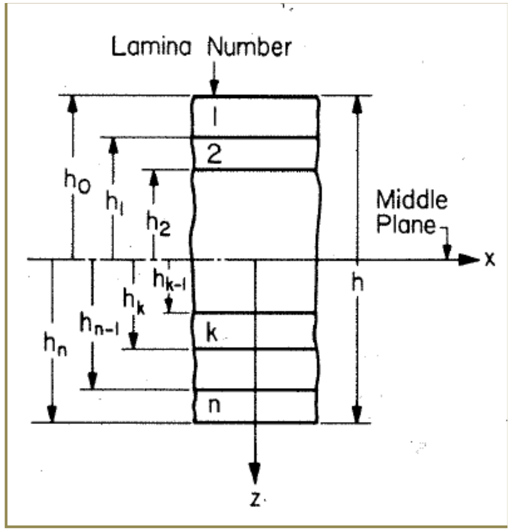
If there is no external loading, the resultant forces {N} and moments {M} are such that

{N}=0 {M}=0 (hygrothermal loads are self-equilibrated)

$$\{N\} = \int_{-h/2}^{h/2} \{\sigma\}\, dz = 0$$

$$\{M\} = \int_{-h/2}^{h/2} \{\sigma\} z\, dz = 0$$

$$\left\{\begin{array}{c} N \\ M \end{array}\right\} = \left[\begin{array}{cc} A & B \\ B & D \end{array}\right]\left\{\begin{array}{c} \varepsilon^0 \\ k \end{array}\right\} - \left\{\begin{array}{c} N^T \\ M^T \end{array}\right\} - \left\{\begin{array}{c} N^H \\ M^H \end{array}\right\} = 0$$



The global deformations are solution of:

$$\left[\begin{array}{cc} A & B \\ B & D \end{array}\right]\left\{\begin{array}{c} \varepsilon^0 \\ k \end{array}\right\} = \left\{\begin{array}{c} N^T \\ M^T \end{array}\right\} + \left\{\begin{array}{c} N^H \\ M^H \end{array}\right\}$$

Thermal loads:

$$\{N^T\} = \Delta T \sum_{k=1}^{n} [\bar{Q}]_k \{\alpha\}_k (h_k - h_{k-1})$$

$$\{M^T\} = \frac{\Delta T}{2} \sum_{k=1}^{n} [\bar{Q}]_k \{\alpha\}_k (h_k^2 - h_{k-1}^2)$$

For a **symmetric laminate**

B=0

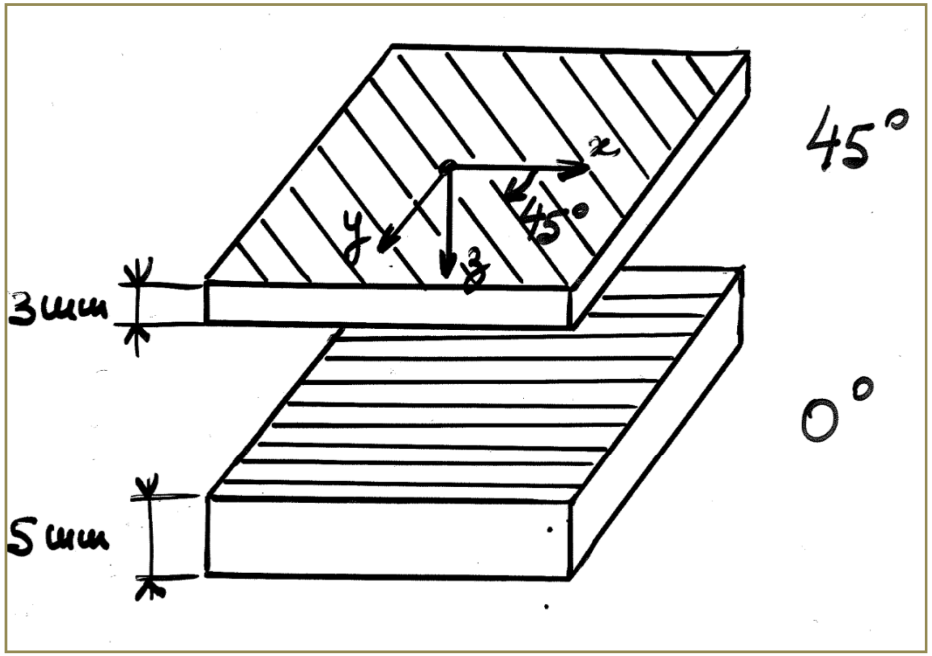
$\{M^T\}=\{M^H\}=0$

Thermal stresses are unavoidable in the fabrication of composites.
The residual stresses due to curing have a significant effect on failure and should not be neglected in the design.
Non-symmetric laminates will experience **warping** during cooling.

Example 6.10: Non-symmetric two-ply laminate (glass-epoxy)

(5mm at 0° and 3mm at 45°) Fabricated at 125°C and cooled at room temperature 25°C

$\alpha_L = 7.0 \times 10^{-6}/°C$ and $\alpha_T = 23 \times 10^{-6}/°C$



Stiffness matrix of one ply in principal material axes:

$$[Q] = \begin{bmatrix} 20 & 0.7 & 0 \\ 0.7 & 2.0 & 0 \\ 0 & 0 & 0.7 \end{bmatrix} \text{GPa}$$

$$[\bar{Q}]_{45°} = \begin{bmatrix} 6.55 & 5.15 & 4.50 \\ 5.15 & 6.55 & 4.50 \\ 4.50 & 4.50 & 5.15 \end{bmatrix}$$

$$\begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{Bmatrix}_{0°} = \begin{Bmatrix} \alpha_L \\ \alpha_T \\ 0 \end{Bmatrix} = 10^{-6} \begin{Bmatrix} 7 \\ 23 \\ 0 \end{Bmatrix}$$

$$\begin{Bmatrix} \alpha_x \\ \alpha_y \\ \frac{1}{2}\alpha_{xy} \end{Bmatrix}_{45°} = \begin{bmatrix} 0.5 & 0.5 & -1 \\ 0.5 & 0.5 & 1 \\ 0.5 & -0.5 & 0 \end{bmatrix} \begin{Bmatrix} 7 \times 10^{-6} \\ 23 \times 10^{-6} \\ 0 \end{Bmatrix} = \begin{Bmatrix} 15 \\ 15 \\ -16 \end{Bmatrix} 10^{-6}$$

Thermal loads:

$$\{N^T\} = \Delta T \sum_{k=1}^{n} [\bar{Q}]_k \{\alpha\}_k (h_k - h_{k-1})$$

$$\{M^T\} = \frac{\Delta T}{2} \sum_{k=1}^{n} [\bar{Q}]_k \{\alpha\}_k (h_k^2 - h_{k-1}^2)$$

$$\Delta T = 25 - 125 = -100°\text{C}$$

$$\Delta T \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix}_{0°} \begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{Bmatrix}_{0°} = 10^{-3} \begin{Bmatrix} -15.61 \\ -5.09 \\ 0 \end{Bmatrix}$$

$$\Delta T \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix}_{45°} \begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{Bmatrix}_{45°} = 10^{-3} \begin{Bmatrix} -10.35 \\ -10.35 \\ -5.26 \end{Bmatrix}$$

Thermal loads:

$$\begin{Bmatrix} N_x^T \\ N_y^T \\ N_{xy}^T \end{Bmatrix} = [(4)-(-1)]10^{-3}\begin{Bmatrix} -15.61 \\ -5.09 \\ 0 \end{Bmatrix} + [(-1)-(-4)]10^{-3}\begin{Bmatrix} -10.35 \\ -10.35 \\ -5.26 \end{Bmatrix} = 10^{-3}\begin{Bmatrix} -109.10 \\ -56.50 \\ -15.78 \end{Bmatrix} \text{GPa} \cdot \text{mm}$$

$$\begin{Bmatrix} M_x^T \\ M_y^T \\ M_{xy}^T \end{Bmatrix} = \tfrac{1}{2}[(4)^2-(-1)^2]10^{-3}\begin{Bmatrix} -15.61 \\ -5.09 \\ 0 \end{Bmatrix} + \tfrac{1}{2}[(-1)^2-(-4)^2]10^{-3}\begin{Bmatrix} -10.35 \\ -10.35 \\ -5.26 \end{Bmatrix} = 10^{-3}\begin{Bmatrix} -39.45 \\ 39.45 \\ 39.45 \end{Bmatrix} \text{GPa} \cdot \text{mm}$$

Thermal moments

Global deformations of the laminate:

$$\begin{Bmatrix} \epsilon^0 \\ \hline k \end{Bmatrix} = \left[\begin{array}{c|c} A' & B' \\ \hline B' & D' \end{array}\right]\begin{Bmatrix} N^T \\ \hline M^T \end{Bmatrix}$$

Non-symmetric laminate

$$\begin{Bmatrix} \epsilon_x^0 \\ \epsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} = 10^{-4}\begin{Bmatrix} -8.14 \\ -20.20 \\ 6.99 \end{Bmatrix} \qquad \begin{Bmatrix} k_x \\ k_y \\ k_{xy} \end{Bmatrix} = 10^{-4}\begin{Bmatrix} 0.58 \\ -1.00 \\ -2.35 \end{Bmatrix}$$

Warping !!

For each ply,

$$\begin{Bmatrix} \varepsilon_x^M \\ \varepsilon_y^M \\ \gamma_{xy}^M \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 + zk_x \\ \varepsilon_y^0 + zk_y \\ \gamma_{xy}^0 + zk_{xy} \end{Bmatrix} - \begin{Bmatrix} \alpha_x \Delta T \\ \alpha_y \Delta T \\ \alpha_{xy} \Delta T \end{Bmatrix}$$

$$\begin{Bmatrix} \epsilon_x^M \\ \epsilon_y^M \\ \gamma_{xy}^M \end{Bmatrix}_{0°} = 10^{-4} \begin{Bmatrix} -8.14 + 0.58z + 7.0 \\ -20.20 - 1.00z + 23.0 \\ 6.99 - 2.35z + 0 \end{Bmatrix} = 10^{-4} \begin{Bmatrix} -1.14 + 0.58z \\ 2.80 - 1.00z \\ 6.99 - 2.35z \end{Bmatrix}$$

$$\begin{Bmatrix} \epsilon_x^M \\ \epsilon_y^M \\ \gamma_{xy}^M \end{Bmatrix}_{45°} = 10^{-4} \begin{Bmatrix} -8.14 + 0.58z + 15.0 \\ -20.20 - 1.00z + 15.0 \\ 6.99 - 2.35z - 16.0 \end{Bmatrix} = 10^{-4} \begin{Bmatrix} 6.86 + 0.58z \\ -5.20 - 1.00z \\ -9.01 - 2.35z \end{Bmatrix}$$

Same for all plies

Stresses in the 0° ply

0° ply, z = 4

$$\begin{Bmatrix} \epsilon_x^M \\ \epsilon_y^M \\ \gamma_{xy}^M \end{Bmatrix} = 10^{-4}\begin{Bmatrix} 1.18 \\ -1.20 \\ -2.41 \end{Bmatrix}$$

$$\begin{Bmatrix} \sigma_x^T \\ \sigma_y^T \\ \tau_{xy}^T \end{Bmatrix} = 10^{-4}\begin{bmatrix} 20 & 0.7 & 0 \\ 0.7 & 2 & 0 \\ 0 & 0 & 0.7 \end{bmatrix}\begin{Bmatrix} 1.18 \\ -1.20 \\ -2.41 \end{Bmatrix} \text{GPa} = \begin{Bmatrix} \sigma_L^T \\ \sigma_T^T \\ \tau_{LT}^T \end{Bmatrix} = \begin{Bmatrix} 2.28 \\ -0.16 \\ -0.17 \end{Bmatrix} \text{MPa}$$

0° ply, z = −1

$$\begin{Bmatrix} \epsilon_x^M \\ \epsilon_y^M \\ \gamma_{xy}^M \end{Bmatrix} = 10^{-4}\begin{Bmatrix} -1.72 \\ 3.80 \\ 9.34 \end{Bmatrix}$$

$$\begin{Bmatrix} \sigma_x^T \\ \sigma_y^T \\ \tau_{xy}^T \end{Bmatrix} = 10^{-4}\begin{bmatrix} 20 & 0.7 & 0 \\ 0.7 & 2 & 0 \\ 0 & 0 & 0.7 \end{bmatrix}\begin{Bmatrix} -1.72 \\ 3.80 \\ 9.34 \end{Bmatrix} \text{GPa} = \begin{Bmatrix} \sigma_L^T \\ \sigma_T^T \\ \tau_{LT}^T \end{Bmatrix} = \begin{Bmatrix} -3.17 \\ 0.64 \\ 0.65 \end{Bmatrix} \text{MPa}$$

45° ply, z = −1

$$\begin{Bmatrix} \epsilon_x^M \\ \epsilon_y^M \\ \gamma_{xy}^M \end{Bmatrix} = 10^{-4}\begin{Bmatrix} 6.28 \\ -4.20 \\ -6.66 \end{Bmatrix}$$

$$\begin{Bmatrix} \sigma_x^T \\ \sigma_y^T \\ \tau_{xy}^T \end{Bmatrix} = 10^{-4}\begin{bmatrix} 6.55 & 5.15 & 4.50 \\ 5.15 & 6.55 & 4.50 \\ 4.50 & 4.50 & 5.15 \end{bmatrix}\begin{Bmatrix} 6.28 \\ -4.20 \\ -6.66 \end{Bmatrix} = 10^{-3}\begin{Bmatrix} -1.05 \\ -2.51 \\ -2.49 \end{Bmatrix} \text{GPa}$$

$$\begin{Bmatrix} \sigma_L^T \\ \sigma_T^T \\ \tau_{LT}^T \end{Bmatrix} = \begin{bmatrix} 0.5 & 0.5 & 1 \\ 0.5 & 0.5 & -1 \\ -0.5 & 0.5 & 0 \end{bmatrix}\begin{Bmatrix} -1.05 \\ -2.51 \\ -2.49 \end{Bmatrix} = \begin{Bmatrix} -4.27 \\ 0.71 \\ -0.73 \end{Bmatrix} \text{MPa}$$

45° ply, z = −4

$$\begin{Bmatrix} \epsilon_x^M \\ \epsilon_y^M \\ \gamma_{xy}^M \end{Bmatrix} = 10^{-4}\begin{Bmatrix} 4.54 \\ -1.20 \\ 0.39 \end{Bmatrix}$$

$$\begin{Bmatrix} \sigma_x^T \\ \sigma_y^T \\ \tau_{xy}^T \end{Bmatrix} = 10^{-4}\begin{bmatrix} 6.55 & 5.15 & 4.50 \\ 5.15 & 6.55 & 4.50 \\ 4.50 & 4.50 & 5.15 \end{bmatrix}\begin{Bmatrix} 4.54 \\ -1.20 \\ 0.39 \end{Bmatrix} = 10^{-3}\begin{Bmatrix} 2.53 \\ 1.73 \\ 1.70 \end{Bmatrix} \text{GPa}$$

$$\begin{Bmatrix} \sigma_L^T \\ \sigma_T^T \\ \tau_{LT}^T \end{Bmatrix} = \begin{bmatrix} 0.5 & 0.5 & 1 \\ 0.5 & 0.5 & -1 \\ -0.5 & 0.5 & 0 \end{bmatrix}\begin{Bmatrix} 2.53 \\ 1.73 \\ 1.70 \end{Bmatrix} = \begin{Bmatrix} 3.83 \\ 0.43 \\ -0.40 \end{Bmatrix} \text{MPa}$$

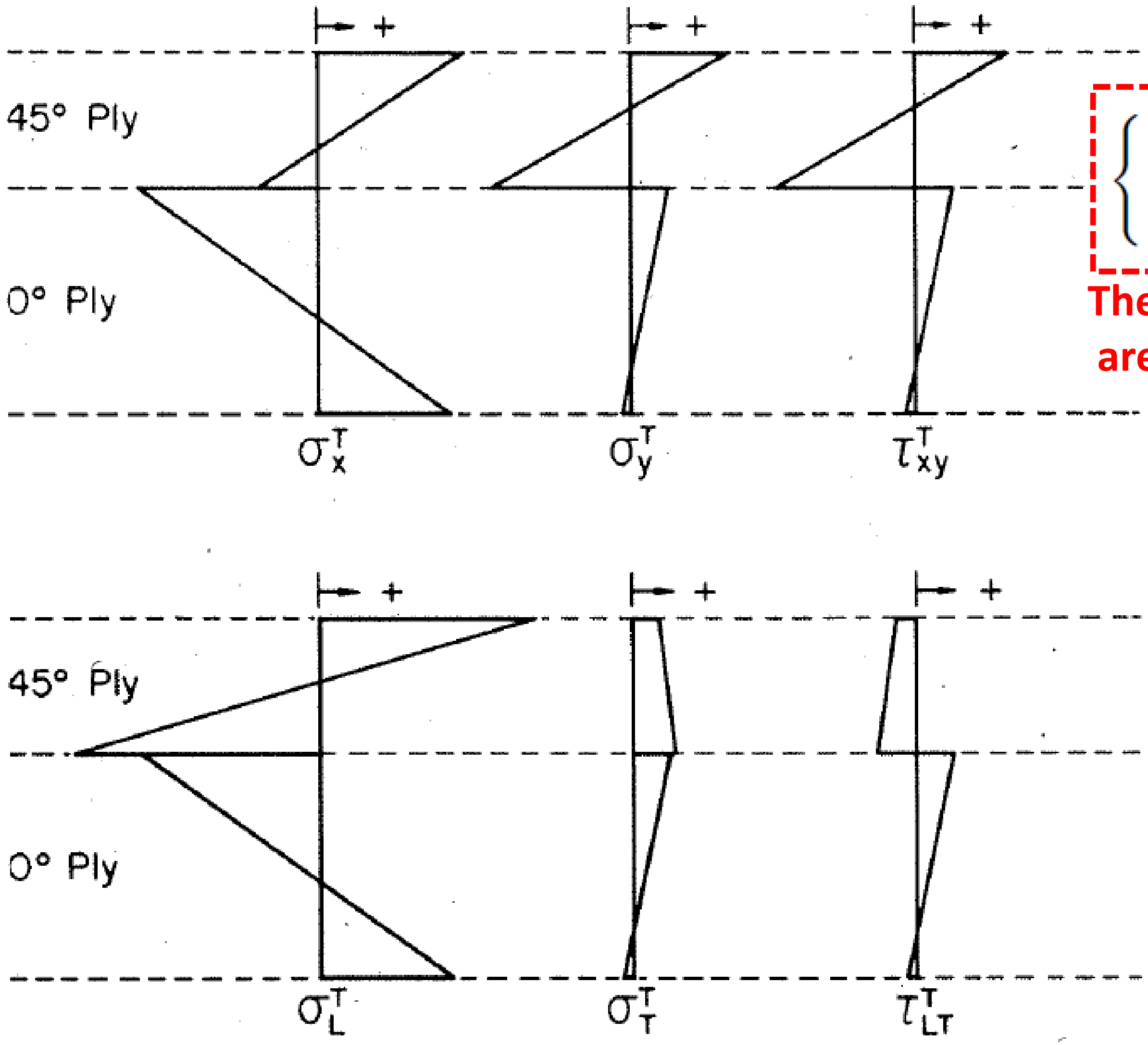
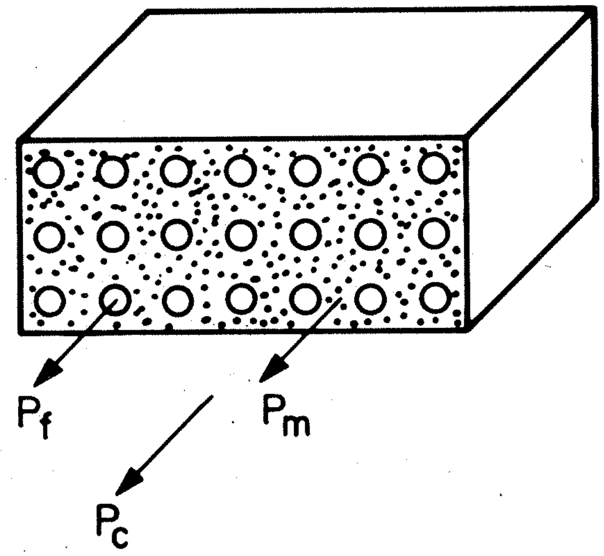


Figure 6.19. Residual stresses (Example 6.10).

$$\left\{ \begin{matrix} N \\ M \end{matrix} \right\}_{xy} = 0$$

The residual stresses are self-equilibrated

Logitudinal CTE (micromechanical model)



1. The fibers and the matrix experience the same strain

$$\epsilon_f = \epsilon_m = \epsilon_c$$

2. The load is shared between the fibers and the matrix

$$P_c = P_f + P_m$$

$$P_c = \sigma_c A_c = \sigma_f A_f + \sigma_m A_m$$

$$\sigma_c = E_c(\varepsilon - \alpha_L \Delta T)$$

$$\sigma_f = E_f(\varepsilon - \alpha_f \Delta T)$$

$$\sigma_m = E_m(\varepsilon - \alpha_m \Delta T)$$

$$E_c = E_f V_f + E_m V_m$$

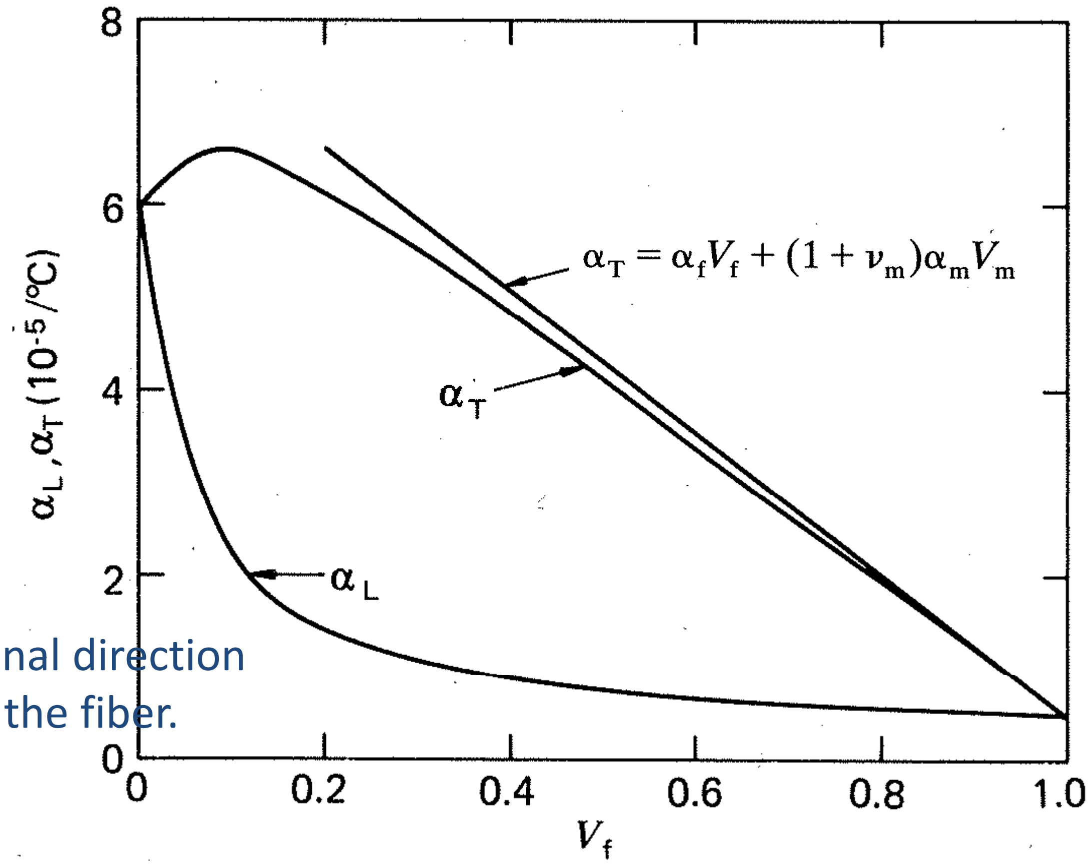
$$\alpha_L = \frac{1}{E_L}(\alpha_f E_f V_f + \alpha_m E_m V_m)$$

Transverse CTE (Schapery): $\alpha_T = (1+\nu_f)\alpha_f V_f + (1+\nu_m)\alpha_m V_m - \alpha_L \nu_{LT}$

Approximation for V_f>0.25: $\alpha_T = \alpha_f V_f + (1+\nu_m)\alpha_m V_m$

Example: Glass-epoxy system

$\alpha_f = 0.5 \times 10^{-5}/°C$	$\alpha_m = 6.0 \times 10^{-5}/°C$
$E_f = 70$ GPa	$E_m = 3.5$ GPa
$\nu_f = 0.20$	$\nu_m = 0.35$



The CTE in longitudinal direction is dominated by the fiber.

Figure 3.32. Coefficients of thermal expansion for uniaxial fiber composite.

Moisture expansion coefficient

1. Isotropic material:

Extension: $\varepsilon = \beta C$

Relative change of volume: $3\varepsilon = 3\beta C$

Moisture content: $C = \dfrac{\text{Mass of water}}{\text{Mass of dry material}} = \dfrac{\varrho_w V_w}{\varrho V}$

Relative volume change: $\dfrac{V_w}{V} = C\dfrac{\varrho}{\varrho_w} = 3\varepsilon = 3\beta C \implies \boxed{\beta = \dfrac{1}{3}\dfrac{\varrho}{\varrho_w}}$

2. Composites:

- Polymer matrices absorb moisture; inorganic fibers do not.
- The expansion in the longitudinal direction is negligible because of the high stiffness of the fibers

$$\beta_L = 0$$

$$\beta_T = \frac{\varrho_c}{\varrho_m}(1 + \nu_m)\beta_m$$

Thermal conductivity

Longitudinal direction (**rule of mixture**) $$k_L = V_f k_f + V_m k_m$$

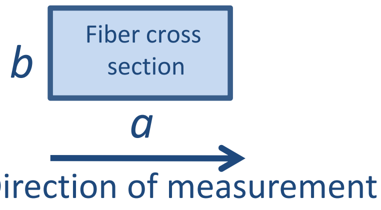
Transverse coefficient may be computed according to **Halpin-Tsai** equation:

$$\frac{k_T}{k_m} = \frac{1 + \xi \eta V_f}{1 - \eta V_f}$$

k_f and k_m refer to the fibers and the matrix in the appropriate direction (fiber anisitropy)

$$\eta = \frac{(k_f / k_m) - 1}{(k_f / k_m) + \xi}$$

$$\log \xi = \sqrt{3} \log \frac{a}{b}$$



For circular fibers, *a=b*

Example: Find k_L and k_T for glass-epoxy and carbon-epoxy for V_f=60% (circular fibers)

$$\text{Epoxy matrix } K_m = 0.25 \text{ W/m/°C}$$

$$\text{Glass fibers } K_f = 1.05 \text{ W/m/°C}$$

Isotropic

$$\text{Carbon fibers } (K_f)_L = 80 \text{ W/m/°C}$$

$$(K_f)_T = 12.5 \text{ W/m/°C}$$

Anisotropic

Glass-epoxy

$$K_L = 0.6 \times 1.05 + 0.4 \times 0.25 = 0.73 \text{ W/m/°C}$$

$$\xi = 1$$

$$\eta = \frac{(1.05/0.25) - 1}{(1.05/0.25) + 1} = 0.615$$

$$\frac{K_T}{K_m} = \frac{1 + 0.615 \times 0.6}{1 - 0.615 \times 0.6} = 2.17$$

$$K_T = 0.543 \text{ W/m/°C}$$

Carbon-epoxy

$$K_L = 0.6 \times 80 + 0.4 \times 0.25 = 48.1 \text{ W/m/°C}$$

$$\eta = \frac{(12.5/0.25) - 1}{(12.5/0.25) + 1} = 0.961$$

$$\frac{K_T}{K_m} = \frac{1 + 0.961 \times 0.6}{1 - 0.961 \times 0.6} = 3.72$$

$$K_T = 0.93 \text{ W/m/°C}$$

Anisotropic !

- Thermal conduction is very anisotropic.
- For carbon epoxy composites, the thermal conductivity of the matrix is negligible.