Chatter reduction through active vibration damping

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Summary and Motivation

Chatter is a problem of instability in the metal cutting process. The phenomenon is characterized by violent vibrations, loud sound and poor quality of surface finish. Chatter causes a reduction of the life of the tool and affects the productivity by interfering with the normal functioning of the machining process. The problem has affected the manufacturing community for quite some time and has been a popular topic for academic and industrial research. The regeneration theory proposed by Tobias and Thusty [1, 2] is referred to by any researcher investigating chatter instability. A feedback model explaining chatter as a closed loop interaction between the structural dynamics and the cutting process was presented by Merrit [3]. The stability lobe diagram is an inseparable part of any study on chatter, since it gives a quantitative idea of the limits of stable machining in terms of two physical parameters, which the machine tool user decides for a machining operation: the width of contact between tool and the workpiece, called the axial width of cut and the speed of rotation of the spindle. Figure 1 shows a typical stability lobe diagram. Merrit [3] showed that the minimum value of the stability limit or the asymptotic level of stability is directly proportional to the structural damping ratio $\xi$ for turning operations. This important finding is the main motivation of the work in this thesis, which proposes active damping to enhance stability limits of the machining operation. A direct consequence of this is an increase in productivity of the machining operation, since higher axial widths of cut imply enhanced metal removal rates. The aim of the thesis is twofold. First, it provides a comprehensive study of regenerative chatter theory for turning and milling. Secondly, active vibration damping, as an efficient chatter suppression strategy is investigated. The organization of the thesis is as follows.

Chapter 1 describes regenerative chatter in the turning process using a simplified proportional cutting force model. The chapter presents a physical explanation of the role of structural damping and the spindle speed on chatter instability. The Root Locus technique is used to complement the physics with a control engineering perspective.

Chapter 2 reviews regenerative chatter in the milling process. The stability analysis is more complicated in comparison to turning, since milling is associated with a rotating cutter and multiple teeth simultaneously cutting the workpiece. The governing equation for regenerative chatter in milling is a periodic delay differential equation, which cannot be analyzed directly by frequency domain techniques. Therefore, time domain simulations are extensively used to demonstrate various aspects of chatter instability in milling. The influence of various physical parameters, such as the type of milling operation, the feed
direction and changes in the structural flexibility on the stability of milling are investigated.

Chapter 3 discusses about existing methods of stabilization of chatter and introduces active damping as the adopted chatter control strategy in the present work. Active damping has favorable features in terms of easy implementation and robustness, if a collocated sensor and actuator configuration is adopted. The effects of active damping on turning and milling are investigated by numerical simulations.

Experimental characterization of chatter may be difficult in a real machining environment, due to the involvement of several parameters, necessity of a large number of machining tests and problems of repeatability of the experimental results. However, the regeneration process and the closed loop representation of chatter are well accepted theories in machine tool chatter research. An alternative way of experimentally demonstrating chatter in a laboratory environment, without conducting actual cutting tests is the subject of study in Chapters 4 and 5. Two mechatronic "Hardware in the Loop" simulators for chatter in turning and milling are presented, which simulate regenerative chatter experimentally without conducting real cutting tests. The development of the demonstrators is a part of the European Union funded SMARTOOL project, intended to propose "smart" chatter control technologies in machining operations. The demonstrators are also used as test beds to investigate the efficiency of active damping, as a potential chatter stabilization strategy.
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Chapter 1
Chatter in turning

1.1 A review of the cutting mechanics

A simple geometry of turning is demonstrated in Figure 1.1, where a tool is cutting a cylindrical workpiece. The metal is removed by a combination of the horizontal penetration $h_0$ of the tool into the workpiece, called the feed and the rotation of the workpiece. The feed is measured by the distance the tool penetrates per revolution of the workpiece (i.e., mm/revolution). The width of the strip of metal removed is called the axial width of cut $a$, which is one of the most important parameters in the stability of the cutting process. Two forces are acting on the tool during the turning operation; force $F_c$ is the radial thrust force and $F_t$ is the tangential component.

![Figure 1.1: A simple turning model](image)

The forces at the contact region are investigated in Figure 1.2. The cutting face, also called the rake face, makes an angle $\alpha$ with the normal and is called the angle of rake. The other face of the tool is the clearance face. $h_0$ is the constant feed of the tool into the workpiece. The formation of the chip is by shear failure of the metal along the plane...
AB. The angle of this plane, relative to the horizontal, is the angle of shear $\phi$. Merchant [4] presented the kinematics of the cutting process, using the representation in Figure 1.2. The chip is considered to be in equilibrium by a system of forces, whose resultant is $P$. $P$ has two components, the frictional force $F_f$ and the normal force $F_n$ between the chip and the tool face. The shear force along the plane AB is $F_s$ and the normal force is $F_n$. $P$ can be resolved also into the normal component $F_c$ and the tangential component $F_t$. Merchant derived relationships between the forces and the cutting parameters $\phi$, $\alpha$, the coefficient of friction $\mu$ between the tool and the chip and the shear strength of the material $\tau$. The relationship is valid in the steady state cutting process, when the forces do not vary dynamically. However, metal cutting is a dynamic process and chatter causes serious problems in the stability. Knight [5] has performed extensive experimental studies on influence of the angle of rake, the feed and cutting velocity on the dynamic stability of the cutting process, following the basic mechanical model of chip formation by Merchant [4].

![Figure 1.2: Merchant’s model of metal cutting](image)

The dynamics of the flexible cutting tool and the workpiece affects the cutting process. While cutting, the tool might face a hard spot on the metal surface and start to oscillate. A wavy surface is left behind on the workpiece surface. After one full rotation, the tool faces the waves left during the previous pass, as shown in Figure 1.3. Let $T = 60/N$ be the time of one full rotation of the workpiece, where $N$ is the spindle speed. Assuming that the tool is flexible in the $Y$ direction and $y(t)$ be the current vibration and $y(t - T)$ be the vibration during the previous vibration, the resultant chip thickness $h(t)$, according to Figure 1.3, is given by Equ. 1.1.

$$h(t) = h_0 + y(t - T) - y$$

(1.1)
Figure 1.3: The regeneration process

The most simple assumption is that the cutting force is proportional to the frontal area of the chip, which is the product of the chip thickness and the width of the cut $a$, as shown in Equ. 1.2.

$$F_c(t) = K_f a [h_0 + y(t - T) - y]$$  \hspace{1cm} (1.2)

where $K_f$ is the cutting coefficient. The phenomenon of the previous and current oscillations of the tool, affecting the chip thickness and the cutting force, is called the ”Regeneration” process. This is the most common reason behind the phenomenon of chatter instability in the metal cutting process. Tobias and Fishwick [1] and almost at the same time Thusty and Polacek [2] independently proposed the phenomenon of regeneration as the reason behind chatter instability. Tobias et al [1] had presented a general expression relating the cutting force $P$ to the various cutting parameters, such as, the instantaneous chip thickness $h_0$, the feed $r$ and the tangential velocity, which is proportional to the angular velocity $\Omega = 2\pi N/60$. The oscillation of the tool-workpiece system generates a perturbation $dP$ in the steady state cutting force $P$. The perturbation is expressed as a function of the increments in the chip thickness $ds$, the feed rate $dr$ and a change in the angular speed $d\Omega$. Therefore, the dynamic cutting force is expressed as,

$$dP = k_1 a ds + \frac{2\pi}{\Omega} k_2 dr + k_3 d\Omega$$  \hspace{1cm} (1.3)
where, $ds = \text{change in the chip thickness}$, $dr = \text{change in the feed rate}$, $d\Omega = \text{incremental change in the spindle speed}$ and $k_1$, $k_2$ and $k_3$ are the corresponding force coefficients. The force coefficients are determined from experimental cutting force data, which provide relationships between the cutting force and each of the parameters. The change in the chip thickness is due to the regeneration effect, so that $ds = y(t - T) - y(t)$. The change in the feed rate is equal to the velocity of the tool $dy/dt$, due to the oscillation. Assuming that the spindle speed remains constant, i.e., $d\Omega = 0$ and substituting $k_1 = K_f$, Equ. 1.3 can be modified into the following form in Equ. 1.4 ([6]).

$$dP = K_f \cdot a \cdot [y(t - t) - y(t)] - \frac{2\pi k_2}{\Omega} \frac{dy}{dt}$$

The second term is negative since velocity in the positive $Y$ direction implies an upward motion of the tool, which reduces the chip thickness and the cutting force $P$. Since the term is proportional to the velocity of the tool, the change in the feed rate appears as a viscous force in the expression of the cutting force. This acts like a damping force in the cutting force model, which influences the stability of the machining process. The effect of the second term is significant for low spindle speeds and this explains the high stability at low spindle speeds, as reported by various authors. For higher $\Omega$, the effect vanishes. The force equation consists of two parts: the first part is in phase with the chip thickness; the second part, being proportional to the velocity, is at quadrature to the displacement. This generates a phase difference between the chip thickness and the cutting force signal as reported by Doi et al [7] and Kegg [8].

The phase difference between the chip thickness and the cutting force and the formulation in Equ. 1.4 point to the existence of damping in the cutting process. To study the dependence of this damping on the frequency of the oscillation, many authors have used frequency domain terminology to relate the two quantities. Nigm [9] related the cutting force and the chip thickness through a complex transfer function to incorporate the phase difference between the quantities. Many authors have worked to generate transfer functions which explain the process damping and are inclusive of all parameters affecting the cutting process. Das et al [10] use separate transfer functions to relate the cutting force and the inner and outer modulations, i.e., $y(t)$ and $y(t - T)$ respectively, which are experimentally measured by Peters et al [11]. Tlusty summarized the work of several authors on this subject in [12]. He proposed the "Dynamic Cutting Force Coefficient Approach" in order to model the damping in the cutting process. The current oscillation of the tool generates a wavy surface on the workpiece, which is termed as "Wave Cutting". The tool removes the undulations, left during the previous pass and this is named "Wave Removal". The normal and the tangential components of the cutting force are related through complex valued transfer functions to the inner and outer modulations, as shown in Equ. 1.5.

$$F_c = a \cdot [K_{di} \cdot y(t) + K_{do} \cdot y(t - T)]$$
$$F_t = a \cdot [K_{ci} \cdot y(t) + K_{co} \cdot y(t - T)]$$

$K_{di}$ and $K_{do}$ are the direct transfer functions, relating the inner and outer modulations to the normal component of the cutting force. Similarly $K_{ci}$ and $K_{co}$ are the cross transfer
functions, relating the tangential component to the same quantities. It is shown analytically and experimentally in [12] that the real parts of $K_{do}$ and $K_{co}$ and imaginary parts of $K_{di}$ and $K_{ci}$ play a role in the stability of the machining process. The damping in the cutting process arises mainly from $\text{Im}(K_{di})$. The work is an advancement in the modeling of cutting forces and it limits the investigation to finding only the inner modulation coefficients for characterization of the cutting process. A physical explanation behind the cause of damping in the cutting is now presented, following the works of Kegg [8] and Thusty [13].

![Diagram](image)

Figure 1.4: Rubbing of the tool flank with the workpiece surface for sharp tool and a) high spindle speed b) low spindle speed c) effect of blunt tool

Figures 1.4 a) and b) compares two situations of cutting at a high and a low spindle speed respectively. Assuming that the tool is oscillating at the same frequency, the number of waves created by the tool during one complete rotation will be lower in case a) in comparison to the slower spindle speed of case b). The waves on the workpiece surface would also be steeper in case b) than in case a). Thus there is a higher possibility of clearance face of the tool, rubbing against the workpiece surface and thereby dissipating energy. This explains a higher stability for low spindle speed machining. The effect is even more dominant when a worn tool is used, as shown in Figure 1.4 c), since the possibility of interference between the tool and the waves on the workpiece increase in this situation.

In the present study, the simplified model of the cutting force is adopted. The cutting force is assumed to be proportional to the regenerative chip thickness. This linear cutting force model is followed in all subsequent theoretical developments in the thesis.
1.2 Stability analysis of chatter

This section deals with the physical aspects of chatter instability. It starts with the classical stability analysis technique and discusses about the physical relationship between the instability, damping and the shape of the chip at different spindle speeds.

1.2.1 The classical stability analysis technique

Referring to Equ. 1.2 and assuming that the machine tool structure is flexible only in the Y direction, the dynamic equation of motion can be written as,

\[ m\ddot{y} + c\dot{y} + ky = K_{f.a.}[h_0 + y(t - T) - y] \] (1.6)

Equ. 1.6 is a time invariant Delay Differential equation (DDE). In Laplace domain \( y(t - T) = y(s) e^{-sT} \). Defining the machine-tool transfer function between the applied force \( F \) and displacement \( y \) as \( G(s) \) and substituting for \( y(t - T) \), we have in Laplace domain,

\[ \frac{h(s)}{h_0(s)} = \frac{1}{1 + K_{f.a}.G(s)(1 - e^{-sT})} \] (1.7)

where

\[ G(s) = \frac{y(s)}{F(s)} = \frac{1}{ms^2 + cs + k} \] (1.8)

Therefore the characteristic equation of the closed loop system is

\[ 1 + K_{cut}.G(s)(1 - e^{-sT}) = 0 \] (1.9)

where \( K_{cut} = K_{f.a} \). From Equ. 1.9 \( K_{cut} \) can be derived as

\[ K_{cut} = \frac{-1}{G(s)(1 - e^{-sT})} \] (1.10)

Equ. 1.9 is not restricted to a single degree of freedom (SDOF) oscillator but can also be extended to single input single output (SISO) systems with multiple degrees of freedom, provided the appropriate expression for \( G(s) \) is used. Merrit [3] introduced a closed loop feedback diagram for regenerative chatter, as shown in Figure 1.5 and is credited for analyzing the phenomenon from a control engineering perspective. Under certain combinations of \( K_{cut} \) and spindle speed \( N \), the feedback loop becomes unstable, leading to chatter.

Equ. 1.10 shows that the stability limit can be derived from a knowledge of the frequency response function \( G(s) \) as proposed by Tlusty et al [2], Merrit [3], Tobias et al [14] and Altintas [15].
Figure 1.5: Merrit’s closed loop representation of chatter

Figure 1.6: Nyquist plot of the system transfer function $G(s)$
Assuming that the system is at the stability limit and oscillating harmonically with chatter frequency $\omega_c$, $s = j\omega_c$ is substituted in Equ. 1.9. Equating the real and imaginary parts to zero and with some mathematical manipulation, the following relationships are obtained.

$$K_{lim} = \frac{-1}{2 \text{Re}(G(j\omega_c))}$$

(1.11)

$$\omega_c T = 2p\pi - 2\tan^{-1}\left(\frac{\text{Re}(G(j\omega_c))}{\text{Im}(G(j\omega_c))}\right) = 2p\pi - \epsilon$$

(1.12)

where $p = 0, 1, \ldots$ and $\epsilon = 2\tan^{-1}\left(\frac{\text{Re}(G(j\omega_c))}{\text{Im}(G(j\omega_c))}\right)$.

$K_{lim}$ is inversely proportional to $\text{Re}(G(j\omega_c))$.

This implies that a stiffer structure will have a higher stability limit. Since stability limit is a physical quantity and is positive, Equ. 1.11 for a SDOF system is valid for values of $\omega_c$, higher than the natural frequency of the machine tool structure, where $\text{Re}(G(j\omega_c))$ is less than zero. This proves that chatter frequencies should be higher than the natural frequency in a SDOF turning operation.

Considering Eqs. 1.9 and 1.10, a graphical method of stability analysis via the Nyquist plot is proposed by Tobias et al [14]. Figure 1.6 is the Nyquist plot of the transfer function $G(s)$. $\overrightarrow{AX}$ represents the frequency response at a certain chatter frequency $\omega_c$, which is slightly higher than the natural frequency. Now from Equ. 1.12,

$$G(j\omega_c)e^{-j\omega_c T} = G(j\omega_c)e^{j(-2p\pi+\epsilon)}$$

(1.13)

Assuming that angles, measured anticlockwise, are positive, $G(j\omega_c)e^{j(-2p\pi+\epsilon)}$ can be represented by the vector $\overrightarrow{AY}$ rotated anticlockwise by $\epsilon$ with respect to the vector $\overrightarrow{AX}$. From Equ. 1.12, $\epsilon = 2\tan^{-1}\frac{\overrightarrow{BX}}{\overrightarrow{AB}}$, where $\overrightarrow{BX}$ and $\overrightarrow{AB}$ are the real and imaginary parts of $G(j\omega_c)$. Therefore $\epsilon$ is equal to twice the angle between $\overrightarrow{AX}$ and the imaginary axis, thereby making $\overrightarrow{AX}$ and $\overrightarrow{AY}$ equal vectors, symmetric about the imaginary axis. The difference between the two vectors, which is the vector $\overrightarrow{YX}$ is a real quantity and equal to twice the real part of $G(j\omega_c)$ and gives the limiting value of $K_{cut}$. The limiting $K_{cut}$ can be increased by making the real part of $G(j\omega_c)$ less negative.
Physically $\omega_c T$ is the total angular displacement of the oscillating tool, vibrating with frequency $\omega_c$ during one period of revolution $T$. $p$ is the number of complete waves traversed by the tool. Therefore, $\omega_c T$ is directly related to the phase difference between successive undulations on the workpiece surface, as shown in Figure 1.7.
Figure 1.8: a) Stability lobe diagram b) Chatter frequencies c) Phase difference between successive undulations
The plot of the stability limit in terms of the ratio of $K_{cut}$ and the stiffness of the tool and the chatter frequency, obtained by solving Eqs. 1.11 and 1.12, for various spindle speeds is the traditional stability lobe diagram. An example of a SDOF system, with a natural frequency of 47 Hz and damping ratio $\xi$ of 1% is presented to demonstrate the stability lobe diagram. In Figures 1.8 a), b) and c), various lobes are numbered according to the value of $p$, used in the calculation. One can observe a repetition of the lobes, which arises from the trigonometric nature of Eq. 1.12. There is an overlap between successive lobes at certain spindle speeds. From the viewpoint of stability limit, the lower limit and the corresponding chatter frequency should be considered. Around the intersection points A and B, there is a jump in the chatter frequency and an abrupt change in the phase difference $\omega_c T$. Four representative points at 2700, 2840, 3790 and 8000 RPM are considered and marked in the figures. The stability limits for 2700, 2840 and 8000 RPM are high and almost identical, but the chatter behavior, for these three cases is different. For 2700 and 8000 RPM, the phase $\omega_c T$ is close to an odd multiple of $\pi$. This corresponds to the vector $\vec{Y}_2 \vec{X}_2$ in Figure 1.6, which has a low amplitude and thereby, a high stability limit occurs according to Eq. 1.11. Vector $\vec{Y} \vec{X}$ tends to zero as $\omega_c T$ approaches $2\pi$. The chatter frequency $\omega_c$ and also the spindle speed frequency $1/T$ are nearly equal to the natural frequency. This situation arises for 2840 RPM, where $\omega_c T = 2\pi$. For $\omega_c T = 3\pi/2$, vector $\vec{Y}_1 \vec{X}_1$ has the largest value. Therefore according to Eq. 1.11, the stability limit is the lowest. This happens for 3790 RPM, where $\omega_c T$ is equal to 270 degrees. Therefore, there is a relationship between the stability limit and phase difference between successive modulations of the chip thickness. A physical explanation of this relationship is discussed in the next subsection.

1.2.2 Discussion of the physics behind chatter

In this subsection, the relationship between chatter instability and the physical parameters, such as the structural damping and the phase difference between successive undulations is presented. References are made to figures 1.6 and 1.8, in order to discuss the various points. At the limit of stability, assuming that the system vibrates with a chatter frequency $\omega_c$, Eqn. 1.6 is written in frequency domain by substituting $s = j\omega_c$.

$$(-m\omega_c^2 + j\omega_c k + k)(j\omega_c) = K_{cut}[h_0 - (1 - e^{-j\omega_c T})]Y(j\omega_c)$$

Neglecting the effect of the feed, the dynamic equation of motion can be written as,

$$[-m\omega_c^2 + j\omega_c (c + K_{cut} \frac{\sin \omega_c T}{\omega_c}) + (k + K_{cut} - K_{cut} \cos \omega_c T)]Y(j\omega_c) = 0$$

It is observed in Eqn. 1.15, that the damping and the stiffness of the system in the closed loop are frequency dependent, due to the term $\omega_c T$. Equating the real and the imaginary parts to zero, the values of $K_{cut}$ and $\omega_c$ at the stability limit can be obtained. A direct relationship between the phase term $\omega_c T$, the damping coefficient and the stability
The stability limit is demonstrated by the following equation.

\[ K_{lim} = -c \frac{\omega_c}{\sin \omega_c T} \]  

Equation 1.16 shows that the stability limit is directly proportional to the structural damping coefficient \( c \), a fact also shown in [3]. The effect of enhancement of damping on the stability of machining is taken up in detail in Chapter 3. Considering the expression of stiffness in Equ. 1.15, it can be seen that the chatter frequency will be greater than the natural frequency of the system since the closed loop stiffness is equal to or greater than the original stiffness of the structure. This is in agreement with Figure 1.6, which shows that \( \omega_c T \) varies between \( \pi \) and \( 2\pi \). This ensures a negative value for \( \sin \omega_c T \) and a positive \( K_{lim} \) in Equ. 1.16. For \( \omega_c T \) approaching an odd multiple of \( \pi \), \( K_{lim} \) tends to infinity. High stability limits are observable for 2700 and 8000 RPM in Figure 1.8 a), for which the phase differences are nearly 180 and 540 degrees respectively. In reality, \( \omega_c T \) cannot reach \( \pi \), since it implies an infinite value of \( \omega_c \), according the Nyquist plot in Figure 1.6. An overlap between successive lobes also restricts the stability limit and chatter frequency to finite values. For the phase difference equal to 360 degrees or its even multiple, the stability limit is also infinity, from Equ. 1.16. This occurs for 2840 RPM (Refer to figures 1.8 a), b) and c)), where the spindle speed frequency, \( 1/T \), is close to the natural frequency. In this case also, the exact situation of \( \omega_c T = 2\pi \) cannot be reached, due to the overlapping between successive lobes. This nonetheless explains the high stability for spindle speeds close to this condition. A plot of \( \omega_c / \sin \omega_c T \) is shown in Figure 1.9 a). \( \sin \omega_c T \) has a minimum value of \(-1\), for \( \omega_c T = \frac{3\pi}{2} \). Therefore a low value of \( K_{cut} \) is obtained from Equ. 1.16. This explains the low stability for 3790 RPM, for which the phase difference is equal to \( \frac{3\pi}{2} \). Highly negative values of \( \omega_c / \sin \omega_c T \) are observed for 2700, 2840 and 8000 RPM, which explain high stability. The discussion therefore explains how the phase difference between successive modulations of the chip thickness, governs the stability limits. Physically this also determines the shape of the chip and the three possibilities are illustrated with shades in Figure 1.9 b). In Region 1 in Figure 1.9 b), i.e., for 2840 RPM (left end of a lobe), the chip thickness is constant, due to the successive modulations of the chip thickness being in phase. Thus the structure will not be excited dynamically and ideally there is no possibility of chatter. For Regions 3 and 4 (2700 and 8000 RPM, right ends of lobes 2 and 1), the shape of the chip is highly deformed due to the successive undulations, being out of phase. This implies a strongly dynamic excitation force. However, the closed loop stiffness, depicted by the third term in Equ. 1.15, has a maximum value of \( k + 2K_{cut} \). Therefore, even if the cutting forces are strongly dynamic, the displacements are small. This is demonstrated by time history plots of the force and displacement for 2700 and 2840 RPM in Figure 1.10. For 2840 RPM, the absence of the regenerative effect causes the force to stabilize to a constant value. In the case of 2700 RPM, even though the oscillations in the force values are substantial, the displacement level is almost identical to that of 2840 RPM. The existence of a higher closed loop stiffness explains this behavior. The qualitative difference between the instabilities in
the high stability regions of the stability lobe diagram is thus explained.

Figure 1.9: a) $\omega_c/\sin(\omega_c T)$ for various spindle speeds b) A typical stability lobe diagram
The study in this subsection identifies the role of two important physical parameters, structural damping and the spindle speed on chatter instability. The stability limit is found to have a proportional relationship with the structural damping. The spindle speed is found to influence the phase between the successive modulations in the regeneration phenomenon and affects the damping and the stiffness of the closed loop system. The investigations, undertaken in this section, guide certain principles behind two chatter control strategies, which will be dealt with in Chapter 3.

1.3 Chatter analysis via the Root Locus Method

1.3.1 Description of the method

In this subsection, the Root Locus method is presented to analyze chatter instability. The method provides a control engineering perspective of the phenomenon. Eqn. 1.9 can be viewed as the characteristic equation of a classical closed loop system with unit feedback, as shown in Figure 1.11. $G(s)(1 - e^{-sT})$ is the open loop transfer function and $K_{cut}$ is the feedback gain. The closed loop poles follow the corresponding root locus for increasing $K_{cut}$ and the stability limit is reached when at least a couple of conjugate roots cross the imaginary axis.
Using the Root Locus Method, the migration of the closed loop poles with change in the values of $K_{cut}$ can be tracked in the complex plane and a better insight into the phenomenon can be obtained.

The transcendental part of the open loop transfer function, i.e., the delay term, gives
rise to a system of time invariant Delay Differential Equation with infinite number of roots. It is approximated by Padé Approximation [16]. The quantity $e^{-sT}$ introduces a phase lag proportional to the frequency, which differs from the phase, introduced by the Padé approximation. The difference depends on the order of approximation chosen for the polynomials and value of the quantity $sT$. In Figure 1.12, the variation from actual phase, due to various orders of Padé approximation, is shown.

The maximum value of $sT$, which depends on the frequency $s$ and the maximum value $T$ (i.e., smallest value of $N$), decides the order of the approximation required for an accurate solution of the eigenvalue problem. For multiple degrees of freedom (MDOF) systems, the highest frequency among the modes included in the model of the structure should be considered.

Equ. 1.9 generates two limit cases, depending on the value of $K_{cut}$.

- For $K_{cut} \to 0$, the roots are the poles of $G(s)(1 - e^{-sT})$ which are the poles of $G(s)$ and an infinite number of poles of $(1-e^{-sT})$ at $s = -\infty \pm j(2n\pi/T)$, where $j = \sqrt{-1}$ and $n$ is any integer.

- For cases where $K_{cut} \to \infty$, the roots are the zeros of $G(s)(1 - e^{-sT})$, which are the zeros of $G(s)$ and the infinite number of zeros of $(1 - e^{-sT})$ at $s = \pm j(2n\pi/T)$.

This is discussed by Olgac et al [17]. Figure 1.13 shows the evolution of the poles for a SDOF system. For low values of $K_{cut}$, the pole (denoted by a cross), closest to the
imaginary axis, is a structural pole. The rest of the poles, due to the delay term, ideally should be at infinite distance from the imaginary axis. But due to the approximation of the delay term and a non-zero initial value of $K_{\text{cut}}$, they are seen at finite but large distances from the imaginary axis. With increasing value of $K_{\text{cut}}$, all the roots approach the imaginary axis and cross it. They ultimately converge to the zeros at $s = \pm j(2n\pi/T)$, i.e. $s/2\pi = \pm j(n/T)$ in Hz units, where $n$ is any integer, for very high values of $K_{\text{cut}}$. So the zeros of the system, due to the delay, are at harmonics of the spindle speed frequency. Traditional techniques of chatter analysis generally recognize that instability arises from the structural mode of the system. However it can be said from Figure 1.13, that for certain spindle speeds, there is always a possibility, that the roots due to the delay may cross over to the right side of the imaginary axis before a structural pole does. It can be shown that the stability lobe diagram consists of different regions, where the source of instability is either the structural pole or the delay pole. The SISO system with a natural frequency of 47 Hz and damping ratio $\xi$ of 1% is investigated for this purpose. Figure 1.14 shows the stability lobe and the chatter frequency diagrams, where different regions are distinguished on the basis of the source of instability.
Figure 1.14: Stability lobe diagrams and chatter frequencies showing regions of instability arising from the structural mode and the delay.

In the next subsection, four representative spindle speeds will be considered for the Root Locus analysis to show the various sources of instability.
1.3.2 Review of chatter for chosen spindle speeds

The Root Locus plots for the SISO example are examined for 2700, 2840, 3790 and 8000 RPM. The loci of the eigenvalues are plotted beyond the stability limit to show the direction of migration of the roots. The stable part of the loci is marked with a thin line and the unstable part with a thicker line. It is seen in Figure 1.15, that a reduction of the spindle speed, causes the poles and the zeros, due to the delay, to migrate towards the real axis. The proximity of the zero to the structural pole, determines the length of the locus to instability. A relatively distant location of the zero, for 8000 RPM, in comparison to 3790 RPM, causes a longer locus of the structural pole to instability. This is demonstrated in figures 1.15 a) and b). This explains a higher stability limit and chatter frequency for 8000 RPM. In Figure 1.15 c), for 2840 RPM, the zero is positioned very close above the structural pole and this is almost a pole-zero cancelation. The advancement of the pole towards instability is nullified and a very high value of $K_{cut}$ is required to make the system unstable. The chatter frequency is almost equal to the natural frequency. This is
in agreement with the discussions about the stability lobe diagram in section 1.2. This shows that if the spindle speed frequency, depicted by the position of the delay zero on the imaginary axis, is located close to the structural frequency, the stability limit is very high and the chatter frequency is approximately equal to the natural frequency. For 2700 RPM, the zero migrates to a position below the structural pole. The close proximity of the zero to the pole also has a pole-zero cancelation effect, as in the former case. However, the structural pole remains in the left half of the complex plane and does not contribute to instability. The instability arises from the delay pole and a very high value of $K_{cut}$ is necessary for its migration from infinity to the imaginary axis. This also explains a jump in the chatter frequency in Figure 1.14 between 2700 and 2840 RPM. At an intermediate speed between these two values, the system may oscillate with two frequencies simultaneously, indicating two roots becoming unstable at the same time [17]. The reason behind very high chatter frequencies at certain spindle speeds can thus be attributed to the instability of the delay pole. The cases of 2700 and 8000 RPM are not qualitatively similar, even though the chatter frequency and stability limit are the same in both cases. For the former, the delay pole is contributing to instability and for the latter, the structural pole is the reason behind instability.

The summary of this subsection is the following. The change in the spindle speed causes the poles and the zeros of the delay to move and this changes the behavior of the root locus and the instability characteristics. The high stability limits for certain spindle speeds (2700 and 2840 RPM) are explained by pole zero cancellation situations and instability due to the delay pole. The chatter frequency for turning is always higher than the natural frequency, since there is no crossing of the imaginary axis at frequencies lower than the natural frequency. High chatter frequencies, at certain spindle speeds, are associated with the delay pole instability. This fact is not very obvious from traditional stability analysis.

1.4 Investigation on a MDOF system

A MDOF (Multiple Degrees of Freedom) structure gives a more realistic idea of the stability aspects of the system in chatter. This section deals with the numerical investigations with the Root Locus Method on a MDOF system to highlight the effect of multiple modes on chatter. There is an important characteristic of the pole zero configuration of a MDOF system. $G(s)$ is the contribution of the mechanical structure in the open loop transfer function of the system. Since it relates the displacement and cutting force at the tool tip, it has the alternating pole zero configuration of collocated systems as discussed by Miu [18] and Preumont [19]. These poles and zeros do not change their positions with change in the spindle speed. This interlacing between poles and the zeros ensures that the structural poles would migrate towards the zeros of $G(s)$ in a closed loop. However as seen in the case of a SDOF system in chatter, a migration of the zeros and poles due to the delay occurs with change in the spindle speed. This also happens in a MDOF system. This disturbs the interlacing property and changes the relative locations of the poles and zeros of the whole
system, which in turn decides whether a structural pole or a pole due to the delay would become unstable. Thus the behavior of a MDOF system under chatter is basically similar to that of a SDOF system except that there are extra poles and zeros coming from the structure. A SISO system with three modes (MDOF machine tool structure) is considered in the present study. The modal properties are shown in Table 1.1.

<table>
<thead>
<tr>
<th>Modes</th>
<th>Frequency (Hz)</th>
<th>Damping (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode 1</td>
<td>47</td>
<td>1</td>
</tr>
<tr>
<td>Mode 2</td>
<td>188.2</td>
<td>1</td>
</tr>
<tr>
<td>Mode 3</td>
<td>423.5</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 1.1: Modal frequencies and damping properties for the MDOF system

The system is studied, using the Root Locus Method for different spindle speeds. The results are summarized in the stability lobe and chatter frequency diagrams in Figure 1.16. The ratio between $K_{cut}$ and the static stiffness is plotted versus the spindle speed. The figure shows that different modes of the structure and the delay contribute to the instability. Four representative spindle speeds are chosen for stability analysis via Root Locus plots. For 7500 RPM, as shown in Figure 1.17 a), the purely imaginary zeros due to the delay are located at multiples of 125 Hz, which is the spindle speed frequency. The complex zeros correspond to the structural zeros. The second mode is becoming unstable in this situation. A reduction in the spindle speed to 5000 RPM changes the source of the instability to the first mode pole as it loops towards the delay zero. This is reflected in the change in the shape of the stability lobe and also a drop in the chatter frequency as shown in Figs. 1.16 a) and b). At 3000 RPM, as shown in Figure 1.17 c), the zero is located at 50 Hz and this is a near pole zero cancelation situation. The instability still comes from the first mode. In Figure 1.17 d), the zero is at the same level as the first mode pole. This proximity of the delay zero to the structural pole hinders the migration towards instability and thus a higher placed delay pole is contributing to instability. A high value of $K_{cut}$ is necessary for the migration of the delay pole to instability. This explains the high stability limit and the situation is similar to that of a SDOF system.
Figure 1.16: a) Stability lobe and b) chatter frequency diagram for the MDOF system

The study shows that the migrating poles and zeros of the delay affect the root locus and the stability characteristics in the MDOF system in a similar way as the SDOF case. The instability may arise from different modes at different spindle speeds and this is reflected in the stability lobes where transition from one lobe to another can be identified from a
change in the shape of the lobe and a jump in the chatter frequency.

1.5 Conclusion

This chapter presents a study of the physics of chatter in turning. The traditional stability analysis technique is reviewed and it is shown that stability depends on two physical parameters: the damping in the system and the phase difference between successive modulations of the chip thickness (This is directly related to the spindle speed and results in different shapes of the chip). Stability limit values are shown to be directly proportional to the damping coefficient of the structure. This is taken up in more detail in Chapter 3. The phase difference between the successive modulations gives rise to different shapes of the chip for different regions of the stability lobe diagram. It is shown that low stability regions have a phase difference between successive modulations of an odd multiple of $\pi/2$. 

Figure 1.17: Locus of eigenvalues for increasing $K_{cut}$ for a) 7500, b) 5000, c) 3000 and 2800 RPM
whereas, for highly stable regions it may be an odd multiple of $\pi$ or an even multiple of $2\pi$. The vibrational behaviors of the system in the latter two situations are found to be qualitatively different, even though the stability limit values may be comparable to each other.

The Root Locus technique is used to present a control engineering perspective of chatter instability. Investigations are performed on SDOF and MDOF systems. It is found that the behavior of the latter is essentially similar to that of a SDOF system, except for the fact that there are more poles and zeros in case of the MDOF structure. A reduction in spindle speed causes a migration of the poles and zeros due to the delay towards the real axis. This affects the locus of the closed loop poles which travel towards instability as $K_{\text{cut}}$ is increased. It is demonstrated that instability can arise not only from the structural pole but also from the so-called delay poles. Thus different regions of the stability lobe diagram can be distinguished on the basis of the mechanism of instability. Traditional stability analysis techniques do not identify instability, arising from the delay poles. It is found that the low stability regions are associated with instability from a structural pole. Certain portions of the high stability regions are associated with instability of a delay pole.
Chapter 2

Chatter in milling

2.1 Introduction

This chapter presents the dynamics of regenerative chatter in the milling process and the effect of various physical parameters on the system stability. Unlike turning, where a single tool is in contact with the workpiece surface, milling involves a rotating cutter with multiple teeth cutting the workpiece. The governing differential equation is a DDE with periodic coefficients. Frequency domain techniques cannot be directly applied unless some approximations are made in the system formulations. Some advanced numerical techniques, developed for stability analysis are summarized briefly in the chapter, as part of the literature survey. Time domain simulations are extensively used to demonstrate the various aspects of chatter. It is demonstrated that regenerative chatter in milling can occur via a Hopf or a Flip Bifurcation. The former is a more common form of instability, whereas the latter appears in low immersion milling operations. The effect of various physical parameters on the stability of the milling process is also investigated. It is observed that the type of milling operation, the direction of feed and changes in the stiffness of the machine tool system affect the stability limits. The developments are demonstrated for a very simple 2 DOF milling tool system. However, the implications can be extrapolated for systems with more complicated structural dynamics and thus certain general guidelines (e.g., choice of feed direction, type of milling operation etc.) for execution of a more stable milling operation can be presented. Structural damping also has an important effect on milling stability similar to turning. However, this is discussed in the following chapter.

2.2 Governing Equations

2.2.1 Cutting Forces

In milling, the metal is removed from the workpiece by a combination of rotation of the milling cutter and horizontal motion of the tool into the workpiece. As shown in Figure 2.1 a), let \( f_t \), called the feed, be the horizontal motion during one rotation of the cutting
tooth. Assuming that the diameter of the tool is much larger compared to the feed, the thickness of the metal encountered by a single tooth can be assumed to be the small strip between the two circular arcs. Then the radial chip chip thickness encountered by the tooth is approximated by,

\[ h_0 = f_t \sin \theta \]  

(2.1)

![Figure 2.1: Horizontal feed in milling](image)

Referring to Figure 2.2, which shows a 2D flexible milling head, the regeneration cutting process, proposed by Tobias [1] and Tlusty [2], can be explained in case of milling. Due to the vibration of the tool, each tooth leaves a wavy surface, which is encountered by the subsequent tooth.
The regeneration is with a time delay equal to the tooth passing period. Assuming that the number of teeth in the milling cutter is \( n \) and the spindle speed is \( N \), the tooth passing period \( T \) is equal to,

\[
T = \frac{60}{nN}
\]  

(2.2)

Referring to Figure 2.2, the \( i \)-th tooth is acted upon by an orthogonal system of forces, \( F_{ti} \), which is the tangential component and \( F_{ri} \), which acts in the radial direction. The force encountered by the milling tooth is taken proportional to the instantaneous chip thickness, as proposed by Koenigsberger et al [20]. Thus

\[
\begin{bmatrix}
    F_{ti} \\
    F_{ri}
\end{bmatrix} =
\begin{bmatrix}
    K_t \cdot h_0 \cdot a \\
    K_r \cdot F_{ti}
\end{bmatrix}
\]  

(2.3)

where \( K_t \) and \( K_r \) are the tangential and the radial cutting constants and \( a \) is the axial width of cut, measured perpendicular to the plane of Figure 2.2. In Equ. 2.3, the forces are calculated assuming the cutter to be with straight flutes. It is evident from the figure, that the direction of the instantaneous tangential and radial forces depend on the location of the milling tooth in the cut. The total force acting on the milling head in the direction of flexibilities, i.e., the \( X \) and \( Y \) directions, is the sum of the contributions of each tooth of the cutter. Early models of milling did not consider the changing direction of the cutting forces. An average value of the cutting force was calculated from the power consumed.
in the cut and the tangential velocity of the tooth. The flexibility of the tool and the workpiece were neglected. An enhanced model of the cutting, incorporating the dynamics of the flexible tool, was developed by Sutherland et al [21]. The model was closest to the regenerative model, which is used in the present study.

The regenerative model is the most popular model for chatter vibrations in milling and is the topic of study in the present work. It follows Merrit’s [3] representation of chatter instability as a closed loop interaction between the cutting forces and the structural dynamics of the system. The equivalent formulation for milling was proposed by Sridhar et al [22]. Considering Figure 2.2, $X$ and $Y$ are the global axes of reference and $\theta$ is the instantaneous position of the $i$-th tooth of the cutter. Let the displacements in the radial and the tangential directions be $u_i$ and $v_i$ respectively. Therefore, referring to Equ. 2.1, the instantaneous uncut radial chip thickness is,

$$h(\theta) = f_t \sin \theta + u_{i-1} - u_i$$  \hfill (2.4)

where $u_i$ and $u_{i-1}$ are the deflections of the $i$-th tooth and the $i-1$ th tooth. Here $u_{i-1} = u(t-T)$ where $T$ is the tooth passing period, as given by Equ. 2.2. Expressing the displacement $u$ in terms of the global displacements $x$ and $y$, the radial chip thickness can be expressed as,

$$h(\theta) = f_t \sin \theta + [ \sin \theta \cos \theta ] \left\{ \begin{array}{c} x(t) - x(t-T) \\ y(t) - y(t-T) \end{array} \right\}$$  \hfill (2.5)

Therefore, referring Equ. 2.3, the tangential and radial cutting forces acting on the $i$-th tooth are expressed as,

$$\begin{bmatrix} F_{ti} \\ F_{ri} \end{bmatrix} = \begin{bmatrix} K_t h(\theta) a \\ K_r K_t h(\theta) a \end{bmatrix}$$  \hfill (2.6)

Transforming to global coordinates, the forces are,

$$\begin{bmatrix} F_{xi} \\ F_{yi} \end{bmatrix} = \begin{bmatrix} -\cos \theta & -\sin \theta \\ \sin \theta & -\cos \theta \end{bmatrix} \begin{bmatrix} F_{ti} \\ F_{ri} \end{bmatrix}$$  \hfill (2.7)

Substituting equations 2.5 and 2.6 in Equ. 2.7, the system of forces, acting on the $i$-th tooth, in the $X$ and $Y$ directions are,

$$\begin{bmatrix} F_{xi} \\ F_{yi} \end{bmatrix} = \begin{bmatrix} -K_t \cos \theta \sin \theta - K_t K_r \sin^2 \theta \\ -K_t \sin^2 \theta - K_t K_r \sin \theta \cos \theta \end{bmatrix} f_t a$$

$$+ \begin{bmatrix} -K_t \sin \theta \cos \theta - K_t K_r \sin^2 \theta \\ K_t \sin^2 \theta - K_t K_r \sin \theta \cos \theta \end{bmatrix} \begin{bmatrix} x(t) - x(t-T) \\ y(t) - y(t-T) \end{bmatrix}$$  \hfill (2.8)

Since milling involves multiple teeth in the cut, the general equation of the milling forces in the two directions is the summation of the contributions from each tooth. This is shown
in Eqn. 2.9. Substituting \( x(t) - x(t - T) = \Delta x \) and \( y(t) - y(t - T) = \Delta y \) and summing up the contributions of \( n \) teeth, the forces in the \( X \) and \( Y \) directions are,

\[
\begin{bmatrix}
F_x \\
F_y
\end{bmatrix} = K_t \cdot a \cdot \sum_{i=1}^{n} \left( \begin{bmatrix}
\alpha_{11i} \\
\alpha_{21i}
\end{bmatrix} \cdot f_t + \begin{bmatrix}
\alpha_{11i} & \alpha_{12i} \\
\alpha_{21i} & \alpha_{22i}
\end{bmatrix} \begin{bmatrix}
\Delta x \\
\Delta y
\end{bmatrix} \right) g(\theta_i) \quad (2.9)
\]

where

\[
\begin{align*}
\alpha_{11i} &= \frac{1}{2} [-\sin 2\theta_i - K_r (1 - \cos 2\theta_i)] \\
\alpha_{11i} &= \frac{1}{2} [- (1 + \cos 2\theta_i) - K_r \sin 2\theta_i] \\
\alpha_{21i} &= \frac{1}{2} [(1 - \cos 2\theta_i) - K_r \sin 2\theta_i] \\
\alpha_{22i} &= \frac{1}{2} [\sin 2\theta_i - K_r (1 + \cos 2\theta_i)] \\
\theta_i &= \theta + (i - 1)2\pi/n \quad (2.10)
\end{align*}
\]

The function \( g(\theta_i) \) denotes whether a tooth is inside the cut or not. \( g(\theta_i) = 1 \) for \( \theta_{\text{entry}} < \theta_i < \theta_{\text{exit}} \) and zero elsewhere, where \( \theta_{\text{entry}} \) and \( \theta_{\text{exit}} \) are the angles of entry and exit respectively. Therefore, expressing the displacement vector as \( \mathbf{x} \) and \( K_{\text{cut}} = K_t \cdot a \), the general form of the forces can be presented as,

\[
\mathbf{F} = \Gamma(t) f_t K_{\text{cut}} + \Psi(t) K_{\text{cut}} (\mathbf{x} - \mathbf{x}(t - T)) \quad (2.11)
\]

\( \Gamma(t) \) and \( \Psi(t) \) are periodic functions with periodicity equal to the tooth passing period \( T \).

The forcing function consists of two components: a periodic component arising out of the feed force and a regenerative component, coming from a periodic modulation of the chip thickness. The stability of the system is determined by the stability of the motions due to the regenerative component of the forcing function.

Milling operations can be identified to be one of the following three types : upmilling, downmilling and slotting. In case of upmilling, the tooth of the cutter traverses through an increasing chip thickness, resulting from the feed. This is shown in Figure 2.3 a), which represents a partial immersion upmilling operation. The word partial immersion implies that the radial depth of cut, as shown in Figure 2.3 a) is a fraction of the cutter diameter. A low or a high immersion operation will therefore be decided by the the difference between \( \theta_{\text{exit}} \) and \( \theta_{\text{entry}} \). A half immersion operation means that the total angle is 90 degrees. In case of downmilling, the tooth of the cutter passes through a progressively reducing chip thickness, as shown in Figure 2.3 b). Full immersion milling or slotting implies that the angle of cut is 180 degrees, as shown in Figure 2.3 c). The tooth starts with zero chip thickness, continues with increasing chip thickness until 90 degrees and then the chip thickness reduces to zero when the tooth leaves the workpiece.

Equ. 2.10 shows that the periodic coefficients depend on the instantaneous angular position of the cutting tooth. The periodic nature of the cutting force coefficients for various immersion conditions is investigated in Figure 2.4. The milling coefficient \( \alpha_{11} \) for a
cutter with 4 teeth, rotating with 6000 RPM is considered. The periodicity is equal to the tooth passing period of 2.5 ms. It is observed that with reduction of the radial depth of cut, the milling coefficient approaches a train of impulses. This is due to the teeth of the cutter remaining in contact with the workpiece for a lesser fraction of a period of revolution. It can be inferred from the figure, that the cutting forces for high immersion milling are more continuous in nature. With reduction in the radial depth of cut, the system is subjected to a train of impulsive cutting forces, as shown for the case with 12.5% radial immersion milling.

Figure 2.3: a) Partial immersion upmilling b) Partial immersion downmilling c) Full immersion milling or slotting
2.2.2 The dynamic equation of motion

Figure 2.2 shows the milling cutter as a flexible system, with the stiffness and damping elements oriented in the \( X \) and \( Y \) directions. Assuming, that the machine tool structure has multiple modes, let \( M, C \) and \( K \) be the generalized \( m \times m \) mass, damping and stiffness matrices of the structure. Assuming \( x \) to be the \( m \times 1 \) displacement vector, the dynamic equation of motion is given by,

\[
M \ddot{x} + C \dot{x} + K x = b F
\]

where \( b \) is the \( m \times 2 \) influence matrix containing the information about the degrees of freedom where the force vector \( F = [F_x, F_y]^T \) is acting. Substituting Equ. 2.11 in Equ. 2.12,

\[
M \ddot{x} + C \dot{x} + K x - f_t \cdot K_{cut} \cdot b \Gamma(t) - K_{cut} \cdot b \Psi(t) \cdot b^T (x - x(t - T)) = 0
\]

In state-space notation, Equ. 2.13 can be written as,

\[
\begin{bmatrix}
\dot{x} \\
\ddot{x}
\end{bmatrix} =
\begin{bmatrix}
0 & I \\
-M^{-1}K & -M^{-1}C
\end{bmatrix}
\begin{bmatrix}
x \\
\dot{x}
\end{bmatrix} +
\begin{bmatrix}
1 \\
0
\end{bmatrix}
\begin{bmatrix}
\Gamma(t) \\
\Psi(t)
\end{bmatrix} +
\begin{bmatrix}
f_t \\
0
\end{bmatrix}
\begin{bmatrix}
x \\
\dot{x}
\end{bmatrix}
\]

Figure 2.4: Variation of the milling coefficient \( \alpha_{11} \) for upmilling and downmilling for various immersions
Equ. 2.14 represents a system of linear delay differential equation (DDE) with periodic coefficients. The solution for $x$ will consist of two parts: a particular solution, due to the forcing function with period $T$, arising from the feed and the solution for the regenerative forcing function, which decides the stability of the whole solution. The first part of the solution has the same periodicity as the tooth pass and therefore the Fourier spectrum of the displacement signal will always contain multiples of the tooth passing frequency. The second part of the solution will decide the frequency of chatter at instability.

### 2.3 Stability Analysis

#### 2.3.1 Mathematical Background

The governing differential equation for a linear time invariant system is,

$$ \dot{q} = Aq $$

(2.15)

where $A$ is a constant matrix. The general solution is of the form $q = e^{A(t-t_0)}q(t_0)$, where $q(t_0)$ denotes the initial condition of the system at time $t_0$. The exponential term is called the state transition matrix $\Phi(t, t_0)$ as it gives information of the state of the system at any time for a given set of initial conditions. For asymptotic stability, the eigenvalues of $A$ should have negative real parts, which means that the eigenvalues of $\Phi(t, t_0)$ should have modulus, less than 1. For time varying systems, there is no general solution of the transition matrix. However, periodic systems, where $A(t+T) = A(t)$, can be analyzed by the Floquet theorem [23]. According to this theorem, the state transition matrix is of the form $\Phi(t, t_0) = P(t, t_0)e^{R(t-t_0)}$ where $P(t + T, t_0) = P(t, t_0)$ and $R$ is a constant matrix. The initial condition is $P(t_0, t_0) = I$. The system is asymptotically stable if the eigenvalues of $R$, also known as the characteristic exponents have negative real parts. A proof of the Floquet theorem is given in Wiberg [24]. $R$ does not have a closed form representation, but it is related to the state transition matrix for only one period, i.e., $\Phi(T, t_0)$ [24], which is also known as the Floquet Transition Matrix or the Monodromy operator as proposed by Insperger [25]. Now $\Phi(T, t_0) = Pe^{R(T-t_0)}$. If the eigenvalues of $R$ have negative real parts, the eigenvalues of $\Phi(T, t_0)$ would have magnitude less than 1. Therefore, if the eigenvalues of the Floquet Transition matrix are located inside the unit circle, the system is asymptotically stable.

This theory can be extended to DDEs with periodic coefficients, e.g., the milling operation. Since the feed term does not contribute to instability, neglecting the feed term, Equ. 2.14 can be written as,

$$ \dot{q} = A_1(t)q(t) + A_2(t)q(t - T) $$

(2.16)

where $A_1(t)$ and $A_2(t)$ are with periodicity equal to the tooth passing period $T$. The state transition matrix for one period, $\Phi(T, t_0)$, is required for stability analysis. But due to the
transcendental nature of the DDE, the matrix is of infinite size and approximations are necessary to obtain a finite sized matrix.

2.3.2 Various methods of stability analysis

The Floquet theorem has been applied by various authors after obtaining a finite sized state transition matrix for one period $T$. Sridhar et al [26] use a result by Hahn [27] and present a graphical method for stability analysis of milling. Recently two methods, have been proposed to obtain a finite sized state transition matrix for one tooth passing period: the Temporal Finite Element Analysis method (TFEA) and the Semi-Discretization Method. The TFEA method is proposed by Bayly et al [28] for low immersion milling for a single toothed cutter in a SDOF milling operation. The formulations are extended to a 2 DOF milling operation by Mann et al [29]. Gabor et al propose the Semi-Discretization method in [30] and present experimental verification in [31] for a SDOF milling operation. The method is extended to the general 2 DOF milling operation by Gradisek et al in [32]. The Matlab based bifurcation analysis toolbox called DDE-Biftool (Engelborghs et al [33]) has recently been applied to the stability analysis of milling by Van Assche et al [34].

Before the advent of these methods, Minis et al [35] and Altintas et al [36] had proposed the expansion of the periodic function $\Psi(t)$ into a Fourier series and approximation of $\Psi(t)$ with the zero-th order coefficient $\Psi_0$. This makes the system time invariant. Substituting $\Psi_0$ for $\Psi(t)$ and neglecting the feed term and Laplace transforming Eqn. 2.14 we get the following equation,

$$sI X(s) = \begin{bmatrix} 0 & I \\ -M^{-1}(K - K_{cut}(1 - e^{-sT})b\Psi_0 b^T) & -M^{-1}C \end{bmatrix} X(s)$$

(2.17)

where $X(s)$ is the Laplace transform of the state vector $[x\dot{x}]^T$. The characteristic equation can be obtained from Eqn. 2.17 as,

$$\det \left( sI - \begin{bmatrix} 0 & I \\ -M^{-1}(K - K_{cut}(1 - e^{-sT})b\Psi_0 b^T) & -M^{-1}C \end{bmatrix} \right) = 0$$

(2.18)

Assuming that the system oscillates with a chatter frequency of $\omega_c$, $s = j\omega_c$ is substituted in Eqn. 2.18. Solving the characteristic equation for the chosen chatter frequency, the limiting $K_{cut}$ and the the tooth passing period $T$ can be obtained. This form of stability analysis is generally called the zero-th order single frequency solution for milling chatter.

2.3.3 Discussion of stability characteristics

From subsection 2.3.1, it is known that at the stability limit the eigenvalues of the Floquet transition matrix lie on the unit circle. Let $\mu$ be the eigenvalue of the Floquet Transition Matrix. At the stability limit, $\mu = e^{j\omega_c T}$, with $\omega_c$ as the chatter frequency. Three cases arise for $|\mu| = 1$. 

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• If \( \mu \) is complex and \(|\mu| = 1\), then the chatter frequencies are given by \( \omega_c \pm \frac{2k\pi}{T} \), where \( k \) is any integer. This implies that due to the exponential term, multiple values exist for the chatter frequency at intervals of the tooth passing frequency \( \omega_T = \frac{2\pi}{T} \). This kind of instability is called the Hopf Bifurcation and is the most common of instability in milling.

• If \( \mu = -1 \), then the chatter frequencies are \( \frac{\pi}{T} \pm \frac{2k\pi}{T} \). This is called Flip Bifurcation and in this kind of instability, the chatter frequency at harmonics of odd multiples one half of the tooth passing frequency.

• For \( \mu = 1 \), the bifurcation is called Saddle Node Bifurcation and it is shown by Davies et al [37], that this case cannot arise in case of milling.

Gabor et al [38] discuss about the existence of multiple chatter frequencies in the milling process; the authors identify three kinds of frequencies: 1. For both stable and unstable milling operations the Fourier spectrum of the displacement signal always contain frequencies at multiples of the tooth passing frequencies. 2. For Hopf Bifurcation cases, the chatter frequency arises close to a modal frequency, along with side bands, existing at intervals of the tooth passing frequency. The chatter frequency is incommensurate with the tooth passing frequency. 3. For Flip Bifurcations, the chatter frequencies are at odd multiples of half of the tooth passing frequency. The TFEA technique and the Semi-Discretization method demonstrate both kinds of bifurcations and characterize them by plots of the evolution of the eigenvalues in the complex plane. The Flip Bifurcation regions manifest as a group of high stability regions in the stability lobe diagram, as observed by Davies et al [37], Gabor et al [39] and Gradisek et al [32].

The zero order single frequency solution proposed in [35] and [36] cannot predict the Flip Bifurcation regions accurately and thus the stability lobes differ from the ones obtained by the TFEA or the Semi Discretization Method. For large immersion milling, the milling forces have a continuous nature and the higher harmonics in the Fourier spectrum of \( \Psi(t) \) may not be of sufficient strength. Thus an average approximation by considering \( \Psi_0 \) is adequate for stability analysis [36]. For low immersion milling, the milling coefficient \( \Psi(t) \) becomes more intermittent. Therefore the higher harmonics in the Fourier spectrum contribute substantially to the force signal.

Figure 2.5 a) demonstrates the time history of the milling coefficient \( \alpha_{11} \) for a near full immersion milling condition at 800 RPM with a 4 teethed tool with \( \theta_{\text{entry}} = 0 \) and \( \theta_{\text{exit}} = 175 \) degrees. For the chosen situation, \( \alpha_{11} \) maintains a value of \(-0.6\) except for a very short period when there is a sudden fall and rise in the value. The zero-th order term of the Fourier expansion is sufficient to approximate the situation, as shown in the figure. With reduction in the radial immersion, an intermittency develops in the milling coefficients, as shown for a 50% immersion upmilling condition in Figure 2.5 b). The averaged coefficient no longer gives a reasonable approximation of the time domain simulation values. Higher ordered Fourier expansions are necessary to approximate the periodic coefficients. A Fourier expansion up to 5 harmonics gives a better approximation of the periodically varying coefficient in comparison to a second order approximation. Therefore,
stability limits obtained with the zero-th order approximation may not be very accurate for low immersion milling operations.

Figure 2.5: Comparison between time history plots of $\alpha_{11}$ with and without Fourier approximation a) High immersion b) 50% immersion upmilling at 800 RPM
Merdol et al [40] include higher order terms in the Fourier expansion and apply the method proposed in [35], [36] for low immersion milling. Flip Bifurcation regions of the stability lobe diagram are better predicted by this method. However, one has to assume a chatter frequency range in this method. Therefore, the qualitative idea of the eigenvalues migrating through the unit circle and leading to different kinds of bifurcations cannot be achieved.

In the present study, the Root Locus Method and time domain simulations are used for stability analysis of chatter in milling. The regenerative periodic closed loop chatter model for milling is shown in Figure 2.6.

![Figure 2.6: Closed loop representation for the 2 DOF milling system](image)

Two approximations are made in the Root Locus method. The delay terms to simulate the regeneration process for the two directions are approximated by the Padé Approximation for eigenvalue analysis, as described in Chapter 1. The periodic term $\Psi$ is approximated by the constant matrix $\Psi_0$. This transforms the problem to a DDE with constant coefficients. For a chosen spindle speed and increasing values of $K_{cut}$, the eigenvalue problem for the system is solved. The value of $K_{cut}$, corresponding to which, a couple of conjugate roots cross the imaginary axis is taken as the stability limit for that spindle speed. The stability lobe diagram is constructed with the values of limiting $K_{cut}$ for various
spindle speeds.

2.4 Time domain simulations: literature survey

This section presents a review of the work done in the field of time domain simulation of chatter in milling. There are certain advantages of time domain techniques over other methods of stability analysis of chatter. The time domain method provides a realistic simulation of the cutting process and chatter instability, since the number of assumptions involved is minimum. The combination of the feed and the relative oscillations between the tool and the workpiece can be incorporated. A qualitative evaluation in terms of severity of the chatter and accuracy of surface finish can be achieved. It provides more insight into the dynamics of the milling operation.

The evolution of time domain simulations from the early simplistic models which ignored the dynamics of the tool and workpiece system and the changing directions of the milling forces to more advanced models is attributed to the efforts of Tlusty and his co-workers. Tlusty et al [41] modeled the changing directions of the cutting forces due to a rotating cutter with helical flutes (teeth arranged in helix) for various cutting conditions. A comparison between the stability limits obtained by the improved formulation of the dynamic cutting forces and from the previous simplified model showed significant differences in Tlusty [42].

A basic non-linearity in the chatter phenomenon is the jumping of the tool out of the cut, due to excessive vibrations. For example, if the tool had lost contact with the workpiece at a certain point, during the subsequent pass of the tool over the same point, the chip thickness calculation involves the current displacement and the displacement, two passes before. This implies that to incorporate instances of the tool losing contact with the workpiece, oscillations during successive passes as well as during earlier passes of the tool on the workpiece need to be considered. Tlusty et al [43] and Sato et al [44] included this non-linearity in a time domain simulation of the milling process and introduced the concept of the multiple regeneration process. Due to the tool losing contact with the workpiece, the excitation forces become zero at instants of loss of contact. This prevents the displacements from growing indefinitely and stabilize them to a certain amplitude.

Time domain simulations have also led to some physical and qualitative analysis of chatter instability. Observations on the relationship between machining stability and structural flexibility are made in [45]. While the stability lobe diagram presents the maximum allowable depth of cut for a chosen spindle speed as the quantitative aspect of chatter, the qualitative aspect of the severity of chatter can be understood from the amplitudes of cutting forces and displacements. Normally this information is not available from the stability lobe diagram. Tlusty et al [46] have proposed a unified graphical approach where both of these aspects are taken into consideration. A set of spindle speeds and axial widths of cut are chosen. Time domain simulations are performed for each value of axial width of cut for various spindle speeds. The maximum peak to peak (PTP) amplitudes of the cutting force for the chosen axial width of cut are plotted versus the spindle speed and joined by
lines. The same procedure is repeated for remaining values of widths of cut, giving rise to a contour map. Thus, for a chosen spindle speed, the limiting axial width of cut corresponds to the maximum among the PTP values. The approach not only gives the stability limit but also a sense of the severity of the chatter process.

A measure of surface accuracy is an important aspect of chatter analysis and work in this direction is undertaken in Altintas et al [47], [48] and Campomanes et al [49]. A refined time domain simulation model is used in the work, which incorporates the combined horizontal feed and rotation of the cutter.

Zhao et al [50] used time domain simulations to compare high and low immersion milling operations. Hopf bifurcation is shown to be the more prominent type of instability in high immersion milling due to domination of regenerative effects. Both Flip and Hopf bifurcations are shown to occur in low immersion milling. The paper also investigates the effect of various physical parameters, such as feed, types of milling operation etc., on milling stability.

The following sections demonstrate various aspects of milling chatter and influence of different physical parameters through time domain simulations. The approach is similar to the investigations performed in [50]. The purpose of the demonstration is of practical importance, since a knowledge of the effect of various parameters on the stability of milling, enables one to plan a more stable milling operation.

### 2.5 Time domain simulation of chatter in milling

#### 2.5.1 Introduction

This section demonstrates the occurrence of Hopf and Flip Bifurcations for some chosen spindle speeds and different milling conditions, i.e., 50% immersion upmilling and down-milling and full immersion slotting by time domain simulations. A 2 DOF milling model is used for that purpose. Several time domain simulations are performed for increasing axial widths of cut and a potential chatter situation is identified from an increasing trend in the amplitude of the displacements. The limit of stability is estimated by identifying the approximate transition point between a stable and an unstable situation. The Fourier Transform of the displacement signal, obtained by running the time domain simulation for a certain length of time is used to generate the Power Spectral Density (PSD) plot of the displacement signal. An inspection of the chatter frequencies in this plot can give an idea of the type of bifurcation. Hopf Bifurcation is associated with a basic chatter frequency near a structural mode. Flip bifurcation manifests as multiple frequencies at odd multiples of one half of the tooth passing frequency. It is observed that the instability may arise from any direction of flexibility and via one of the two above mentioned mechanisms for different spindle speeds. Certain assumptions are made for the time domain simulations and they are as follows. The tool is assumed to be the flexible part of the system. A linear relationship between the cutting force and the chip thickness is assumed, even though non linear models have been used in [30], [38] etc. Non linear behavior of the tool leaving the
The properties of the 2 DOF system are listed in Table 2.1. The $X$ direction is stiffer in comparison to the $Y$ direction. It is assumed that the feed direction coincides with the stiffer $X$ direction.

<table>
<thead>
<tr>
<th>Directions of Flexibility</th>
<th>Stiffness (N/m)</th>
<th>Mass (kg)</th>
<th>Frequency (Hz)</th>
<th>Damping (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X$</td>
<td>142122.3</td>
<td>1</td>
<td>60</td>
<td>1</td>
</tr>
<tr>
<td>$Y$</td>
<td>35530.6</td>
<td>1</td>
<td>30</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 2.1: Modal frequencies and damping properties for the MDOF system

Figure 2.7: Orthogonal orientation of the damping and flexibility elements of the tool

2.5.2 Full immersion milling

The stability analysis of full immersion slotting operation, i.e., $\theta_{\text{entry}} = 0$ and $\theta_{\text{exit}} = 180$ degrees, is first considered. The tool is with four teeth, each oriented at 90 degrees interval. Due to this orientation of the teeth, there will always be two teeth inside the cut. It can be shown from Equ. 2.9 and 2.10, that the feed force and the milling force coefficients are always constant for any location of the teeth inside the cut. Therefore slotting with a 4 teethed tool is a special case, where the milling coefficients remain constant and there is no periodicity in the process. The PSD of the displacements in the $X$ and $Y$ directions for a spindle speed of 500 RPM is shown in Figure 2.8. Due to the non-periodic milling force coefficients, the harmonics at tooth passing frequencies are absent. The chatter frequency is around 30 Hz and this shows that chatter is triggered in the more flexible $Y$ direction. The situation, corresponding to 1200 RPM is shown in Figure 2.9. The PSD plots show that
the chatter frequency apparently arises from the 30 Hz mode because the corresponding amplitude is maximum. However, Figure 2.10 shows that the displacement in the $X$ direction is unstable and that in the $Y$ direction is stable. Therefore, in this situation, the chatter arises from the mode in the $X$ direction at 60 Hz. Due to higher stiffness in the $X$ direction, the amplitude of displacement is lower, even though the situation is unstable.
Figure 2.9: Slotting: PSD of displacement signals for 1200 RPM slotting

Figure 2.10: Time history plots of displacements for 1200 RPM slotting
2.5.3 50% immersion upmilling

In case of full immersion milling, the instability at various spindle speeds is associated with Hopf bifurcation. Both Hopf and Flip bifurcations arise for partial immersion milling and this is investigated for 50% immersion upmilling and downmilling operations. The periodic milling force coefficients and feed forces give rise to a system of forces with periodicity equal to the tooth passing frequency. This is demonstrated for a stable upmilling operation at 1200 RPM in Figures 2.11 and 2.12. The former shows the PSD of the milling forces in the two directions. Since the tool has 4 teeth, the harmonics are located at multiples of the tooth passing frequency of 80 Hz. The spectra of the displacement signals are presented in Figure 2.12. The tooth passing frequency appears at multiples of 80 Hz (marked by circles) and peaks are observable at 60 and 30 Hz in the $X$ and the $Y$ directions respectively.

Figure 2.11: PSD of the milling forces for 50% immersion upmilling for 1200 RPM in a stable situation
Chatter situations for upmilling at three spindle speeds are investigated. The situation for 400 RPM is shown in Figures 2.13 and 2.14. The harmonics of the tooth passing frequency occur at multiples of 26.67 Hz, shown by circles in Figure 2.13. Although the time history plot in Figure 2.14 shows unstable vibrations in both directions, stronger unstable vibrations are observable in the \( Y \) direction. A Hopf bifurcation is occurring at this spindle speed, since the chatter frequency is incommensurate with the tooth passing frequency.
Figure 2.13: PSD plots of displacements in the $X$ and $Y$ directions for 400 RPM upmilling

Figure 2.14: Time history plots of displacements in the $X$ and $Y$ directions for 400 RPM upmilling
Figure 2.15: PSD plots of the displacements in the X and Y directions for 500 RPM upmilling

Figure 2.16: Time history plots of the displacements in the X and Y directions for 500 RPM upmilling
In case of 500 RPM, the tooth passing frequency is 33.33 Hz, as shown in Figure 2.15. Although, the maximum amplitude in the PSD plot is at 30 Hz, time history plots in Figure 2.16 show unstable oscillations in the X direction. A Hopf bifurcation is also occurring at 60 Hz.

![Figure 2.17: PSD of the displacements in the X and Y directions for 750 RPM upmilling](image)

For 750 RPM, a Flip bifurcation is occurring. The chatter frequency occurs at 25 Hz which is one half of the tooth passing frequency of 50 Hz. Odd harmonics of the chatter frequency are visible in the PSD plots for displacements in the two directions, as shown in Figure 2.17.

### 2.5.4 50% immersion downmilling

A similar analysis is performed for downmilling for three spindle speeds. In case of 400 RPM, a Hopf bifurcation is occurring. Strong unstable oscillations occur in the X direction in Figure 2.19, even though the maximum magnitude in the PSD plot occurs in the Y direction.
Figure 2.18: PSD of the displacements in the X and Y directions for 400 RPM downmilling

Figure 2.19: Time history of displacements in the X and Y directions for 400 RPM downmilling
Figure 2.20: PSD of the displacements in the X and Y directions for 750 RPM downmilling

Figure 2.21: PSD of the displacements in the X and Y directions for 1200 RPM downmilling
For 750 RPM, a Hopf bifurcation occurs and the chatter frequency appears around the 30 Hz mode in the $Y$ direction, as shown in Figure 2.20. 1200 RPM is a case of Flip bifurcation, as shown in Figure 2.21.

The examples show that in slotting Hopf bifurcation of the structural modes is the only form of instability that can occur. In case of upmilling and downmilling, both Hopf and Flip bifurcation can occur. The stability lobes at different spindle speeds thus have different sources of instability. This reflects the complexity of chatter in the milling process.

### 2.6 Comments on the stability limits

This section investigates the effect of various physical parameters, such as the type of milling, the direction of feed, changes in the stiffness of the structure on the stability lobe diagram. The stability limits for upmilling and downmilling, obtained from the time domain simulations, are plotted as a ratio between the limiting $K_{cut}$ and the stiffness in the $X$ direction in Figure 2.22. An interesting fact is that the high stability regions of one type of milling operation correspond to the low stability regions of the other type, which is also observed in [50]. So depending on the spindle speed, where one wishes to machine, a potential chatter situation may be avoided by just changing the cutting condition from one form of milling to another [38]. A knowledge of the stability lobe diagram is necessary for that purpose.

![Figure 2.22: Stability lobe diagram for upmilling and downmilling for the 2 DOF milling system](image)

Figure 2.22: Stability lobe diagram for upmilling and downmilling for the 2 DOF milling system
The characterizations of the stability lobes, based on the type of instability, are shown for up and downmilling in Figure 2.23. The time history of the oscillations in the two directions are used to identify which direction is primarily causing the chatter instability. If unstable oscillations in one direction are stronger than the other direction, then it is inferred that the instability is primarily associated with that direction. The PSD of the oscillations of the identified direction is checked. The tooth passing frequencies can be easily separated from the spectra. If the condition is that of a Hopf bifurcation, normally a prominent peak is observable near the modal frequency. Flip bifurcations are easier to identify since the corresponding chatter frequencies occur at odd multiples of one half of the tooth passing frequency. The low stability regions of the stability lobe diagram are found to be associated with Hopf Bifurcation, whereas, certain portions of the high stability regions are Flip Bifurcation areas.
Figure 2.23: Regions of Hopf and Flip bifurcations in the stability lobe diagrams for up and downmilling

A change of the feed direction to the more flexible Y direction changes the stability lobes for upmilling drastically. This is shown in Figure 2.24. Since, the Y direction is more flexible, it is more vulnerable to chatter instability and thus the stability limits are lower in comparison to the values, when the feed is in the stiffer X direction.
Figure 2.24: Comparison between stability lobes for upmilling with feed in the X and Y direction

Figure 2.25: Comparison between stability lobes for original structure and modified structure with reduced stiffness in one direction

The effect of a change in the structural dynamics on the stability lobe diagram is
demonstrated in Figure 2.25. The stability lobes for upmilling for the structure, described in Table 2.1, and a modified structure with 90% of the original stiffness in the X direction, are compared. It is found that a change in stiffness causes the stability lobe diagram for the modified structure to shift horizontally with respect to the stability lobe diagram of the original structure. This has implications on control strategies of chatter. The control system needs to be robust in face of all uncertainties in the instability of the milling process.

Another important parameter controlling the stability of the milling process is damping, which is the main motivation behind this thesis. This parameter is not discussed in this chapter. Chapter 3 is devoted to the detailed study of effect of enhanced damping on chatter instability for both turning and milling.

2.7 Stability analysis with Root Locus Method

The Root Locus Method is used for stability analysis for three different cases of milling: full immersion milling or slotting, 50% immersion upmilling and downmilling. For a chosen value of spindle speed, the method solves the eigenvalue problem of Equ. 2.18 for increasing values of $K_{\text{cut}}$, assuming an average value $\Psi_0$ for the periodic milling coefficient matrix $\Psi$. The delay term is approximated by Padé Approximation. The value of $K_{\text{cut}}$, corresponding to which a couple of conjugate roots cross the imaginary axis is the stability limit for the chosen spindle speed. Figure 2.26 compares the stability limits obtained by time domain simulations and the Root Locus Method for slotting. There is a good match between the results obtained from the two methods.

Figure 2.27 compares the stability limits for 50% immersion upmilling, obtained by the two methods. It is observed that the Root Locus Method is able to predict well the stability limits in the low stability Hopf Bifurcation regions. However, for spindle speeds associated with Flip Bifurcation, the results do not match very well. The same observation is made for the case of downmilling in Figure 2.28.
Figure 2.26: Stability limits for slotting operation obtained by Time Domain simulations and the Root Locus Method

Figure 2.27: Stability limits for upmilling operation obtained by Time Domain simulations and the Root Locus Method
Figure 2.28: Stability limits for downmilling operation obtained by Time Domain simulations and the Root Locus Method

The reason behind the inaccuracies in prediction of the stability limits has been explained in a previous section. The zero-th order matrix $\Psi_0$ cannot approximate the periodic milling coefficients accurately for low immersion milling, leading to inaccuracies in prediction of stability limits, especially in the Flip Bifurcation regions.

2.8 Conclusion

This chapter presents the dynamics of regenerative chatter in the milling process. A simple 2 DOF model of a flexible milling tool is considered for stability analysis by time domain simulations. Two kinds of Bifurcations, i.e., the Hopf and the Flip Bifurcations, reported in literature, are demonstrated via time domain simulations. Slotting is characterized by Hopf bifurcation, whereas both Hopf and Flip bifurcations occur for partial immersion milling operations. Flip bifurcation is found to be associated with certain portions of the high stability region of the stability lobe diagram. Since regenerative chatter in milling is formulated by a periodically varying delay differential equation, ordinary frequency domain techniques cannot be applied for stability analysis. The Root Locus Method is applied in the present study by approximating the periodic milling coefficients by the zero-th order term of the Fourier expansion. It is observed that the Root Locus Method is able to predict the stability limits accurately for full immersion milling where only Hopf Bifurcation occurs. Flip bifurcation regions for low immersion milling are not accurately predicted by the method. The chapter demonstrates the inherent complexities of regenerative chatter in
milling in comparison to chatter in turning. Chatter in milling is associated with governing equations that are periodic, more complicated methods of stability analysis and most importantly multiple mechanisms of instability. The involvement of various parameters on stability lobes is also discussed. The type of milling operation that is undertaken affects the stability lobes. It is observed that for upmilling and downmilling, the high stability regions of upmilling correspond to the low stability regions of downmilling. Milling with the feed direction coinciding with a more flexible direction of the structure is found to be more susceptible to chatter. The effect of damping is not discussed in the present chapter and is taken up in Chapter 3.
Chapter 3

Active Control of Chatter

3.1 Introduction

There have been numerous investigations dealing with control of chatter instability. Chatter control can be categorized into two methodologies; online or active control and offline or passive control techniques. The foremost among the offline techniques is the stability lobe diagram, which gives the limit of the width of cut for a chosen spindle speed, as proposed by Tobias et al in [1] and [51]. However, derivation of stability lobes experimentally may be difficult. One needs to conduct large number of cutting tests to establish these diagrams experimentally. This involves a huge amount of labour and may not be practical in many cases. There may be problems of repeatability due to uncertainties in the machining conditions. An automated technique of identifying the locations of the high and low stability regions of the stability lobe diagram, by monitoring a statistical chatter indicator, calculated from the cutting force signals, has been proposed by Soliman et al [52]. Numerical simulations provide an alternative for generation of approximate stability limit information. Time domain methods are most realistic to simulate chatter though long computations are involved in the process. Analytical methods proposed by Minis et al [35] and Altintas et al [36], [53] are capable of generating stability lobes quite efficiently by using an identified model of the tool point transfer function that is estimated by the impact hammer test procedure or by a new method, proposed by Sims et al [54] for small tools. The Cutpro 7 software [55], developed by the Manufacturing Laboratories Inc, is also in use to generate approximate stability lobe diagrams. The stability lobe diagram thus constitutes an important part in the understanding of chatter and needs to be generated either experimentally or numerically. It also gives an idea of the efficiency of the applied passive or active control setup.

The central idea of this chapter is to present a survey of the various methods of chatter control, followed by the introduction of active damping, which is the adopted chatter control strategy for turning and milling. Having discussed the concept of active damping, the chapter proceeds to investigations on the effect of active damping on the stability of turning and milling operations.
3.2 Control of chatter: A review

Regenerative chatter, as shown by Merrit [3], is due to a closed loop interaction between two independent entities: the machine tool structural dynamics and the dynamics of the cutting process. Any method of chatter suppression tries to influence one of the two entities, so that the ultimate goal of higher stability is achieved. Prominent among the methods of influencing the cutting process is online control of spindle speed. This is effected in two ways, either by the "spindle speed selection" method or by "spindle speed modulation". The spindle speed selection technique is described as follows. Fig. 1.9 b) shows a highly stable machining region existing between regions 1 and 4 of the stability lobe diagram that is also commented on in [56]. In this region, the spindle speed frequency for turning or the tooth passing frequency for milling is equal to the dominant natural frequency of the structure. Changing the spindle speed to the stable part of the stability lobe diagram can stabilize an unstable machining operation. This idea was first put forward by Weck et al [57]; the authors proposed an online method for chatter avoidance in face milling operations. The control unit monitors the frequency content of the vibrations of the cutting tool and identifies if a self-excited chatter vibration component exists in the sensor signal. If a chatter frequency is identified, the chatter control program is invoked, which searches for the closest spindle speed where the stability is the highest. If such a speed is found, a speed change command is sent to the driving motor of the spindle. If no such favorable speed is found, the program commands the reduction of the axial width of cut. The method uses a simplified calculation of the stability lobe diagram from the identified chatter frequency. Since milling operations are associated with changes in the structural resonant properties, due to changing of machine configurations and dimensions of the workpiece, stability lobe diagrams are not unique and are dependent on the machining condition. In that respect, for proper functioning of the algorithm, a knowledge of all possible stability limits is necessary. In order to handle such a situation, the authors propose an adaptive control strategy for changing the feed and the axial depth of cut in the milling operation with an aim of maximum utilization of the capacity of the machine.

Smith et al [58] implemented a spindle speed control algorithm, which is based on the same idea but does not require the stability lobe data. The method involves detection of the dominant chatter frequency by sensing the sound, emanated in the cutting process by a microphone and analyzing its frequency content, as proposed by Delio et al [59]. The cutting force signal, sensed with dynamometers, is usually used for chatter monitoring. Identification of the chatter frequency may be difficult due to the existence of strong tooth passing frequency components in the spectra. A more practical approach is to use audio signals since generation of a loud noise is typical of an unstable milling process. The sensed audio signal should normally contain a distinct peak, corresponding to the chatter frequency. This makes chatter detection more efficient than using a dynamometer. The chatter stabilization procedure is as follows. The new spindle speed is calculated in such a way, so as to equate the tooth passing frequency to the sensed chatter frequency. The spindle speed converges to the most stable region of the stability lobe diagram within two or three iterations. The method does not require the knowledge of the stability lobe
diagram for stabilization of chatter. However, there are some limitations. The technique performs well if there is a single dominant natural frequency of the structure. In reality, more than one structural mode may be involved in chatter. The control strategy works well in the high spindle speed regions, where there are well separated lobes. Convergence may be poor in the low spindle speed regions, where the stability lobes overlap each other and in situations where multiple structural modes contribute to chatter. The method also requires stoppage of machine feed every time the spindle speed is changed. The procedure also requires the chatter instability to be triggered in order to identify it and then take a corrective action. This may be detrimental to the life of the machine tool.

Another popular on-line method for chatter avoidance is the spindle speed modulation technique. This involves a continuous periodic modulation of the spindle speed with a very low frequency. This was demonstrated by Hoshi et al [60] experimentally for turning. Subsequent developments are made by by Sexton et al [61] for turning and Lin et al [62] for face milling. De Canniere et al [63] have contributed to the mathematical analysis of variable speed machining. The technique is however costly and limited by the inertia of the rotating parts of the machine.

Ismail et al [64] address the limitations of the previous two techniques of chatter stabilization; the authors propose a ramping of the spindle speed until the cutting process is stabilized. A statistical indicator is simultaneously calculated from the cutting force signal data to detect the severity of the chatter condition. An earlier work of Ismail and Kubica [65] defines a threshold for this quantity, which approximately indicates the occurrence of chatter. As long as the indicator is above the specified threshold, the spindle speed is ramped up and this is continued until the machining process is stabilized. This technique eliminates the need to stop the machine feed when the change in the spindle speed is carried on. The method requires fast computational resources for simultaneous monitoring of the cutting force signal and online control of the spindle speed.

Use of special tool geometry to control chatter has been proposed by various authors. Slavicek [66] and Vanherck [67] proposed the usage of variable pitch milling cutter in order to affect the regeneration process and thus control chatter. Online control of the tool geometry is also used to suppress chatter. It is well known that an adjustment of the tool clearance and rake angles to cause more rubbing between the tool and the metal surface, results in dissipation of energy and stabilization of chatter. Liu et al [68] demonstrated the effect of different rake and clearance angles on the dynamic stability and proposed the concept of tool geometry control to suppress chatter vibrations. Ehmann et al [69] have proposed a rotating cutting tool for lathe to simultaneously vary the rake and clearance angles to stabilize chatter vibrations. Yang et al [70] propose an oscillating cutter to reduce the amplitude of the vibrations in a turning operation. The idea of active control of tool geometry may be easy to apply on single point of contact turning operations but may not be suitable for multiple teeth milling operations.

Vibration control is another strategy to suppress chatter instability. The aim of this strategy is to reduce the relative displacements between the tool and the workpiece and thus suppress chatter. Dohner et al [71] propose a pole placement strategy using a LQG controller and strain gage sensors and electrostrictive actuators for milling. Smart fluids
such as electrorheological or magnetorheological fluids have been used for chatter suppression in Wang et al [72] and Segalman et al [73].

Nachtigal [74] proposes a feedforward approach for control of chatter in lathe and extend the approach to boring operations in [75]. An adaptive vibration control technique, using filtered-X least mean square algorithm is proposed by Browning et al [76] for chatter suppression in boring bars.

Chatter control through active damping is adopted in the present study. The motivation behind this choice arises from many studies, which emphasize on the relationship between damping and stability of the machining process. Damping arising from the cutting process is found to stabilize chatter, as shown in Tlusty [12]. Merrit [3] has shown that the minimum value of the limiting width of cut is directly proportional to the structural damping ratio $\xi$. This implies that increase in the structural damping would raise the asymptotic threshold of stability, i.e., the minimum value of $K_{cut}$ and the region of stable machining would increase. Structural damping can be augmented either by passive or active means. Hahn [77] proposed the use of Lanchester damper as a passive way of damping chatter vibration. Ema et al [78] proposed the use of an impact damper for increasing the stability in boring bars. Schmitz et al [79] propose a tuning of the natural frequencies of the cutting tool and the spindle and tool holder setup. The aim of this design is to make the spindle and tool holder components behave like a passive vibration absorber, thereby, reducing the vibrations of the tool overhang.

A piezoelectric inertial actuator is used by Tarng et al [80] for passive suppression of chatter in turning; the authors tune the actuator to the natural frequency of the tool structure. Passive vibration damping techniques exhibit advantages of easy implementation, low cost and no need for external energy. Moreover they do not lead the system to instability, while active control methods can make the system unstable. But for good performance, the tuning of the dampers needs to be accurately done, which is difficult, due to uncertainties in the resonant properties of the structure. In case of mismatch, a deterioration of performance of tuned passive dampers is observed.

The following section introduces the concept and various methods of active damping existing in literature. The discussion is continued on velocity feedback strategy, which is used to actively damp chatter in the current work.

### 3.3 Active damping

#### 3.3.1 Collocated and non collocated control

Feedback control of structural vibrations is a topic of extensive research and active damping is one of the popular vibration control techniques. The basic concept of a feedback control system is shown in Figure 3.1 where $G(s)$ represents the system transfer function, $H(s)$ the control system and $g$ the feedback gain. The closed loop transfer function between the
input $r$ and the output $y$ is given by,

$$\frac{y(s)}{r(s)} = \frac{gG(s)H(s)}{1 + gG(s)H(s)}$$  \hspace{1cm} (3.1)\]

The characteristic equation of the system is given by $1 + gG(s)H(s) = 0$. Solution of the characteristic equation gives the location of closed loop poles of the system. If $g = 0$, the resonant frequencies or poles are that of the open loop transfer function $G(s)H(s)$. For $g \rightarrow \infty$, the closed loop poles are the zeros of the open loop transfer function. Thus in the Root Locus plot, the closed loop poles migrate from their open loop positions to the zeros as the value of $g$ is increased. The loci of the closed loop poles depend on the relative positions of the poles and zeros of the open loop transfer function and this has an important effect of the robustness of the closed loop control.

![Figure 3.1: A general feedback control system](image)

The general equation of motion of a SISO system with multiple degrees of freedom is given by the following equation,

$$M\ddot{X} + C\dot{X} + KX = b_1 f$$  \hspace{1cm} (3.2)\]

where $M$, $C$ and $K$ represent the $m \times m$ diagonal mass, damping and stiffness matrices of the structure, $X$ is the $m \times 1$ displacement vector and $b_1$ is the $m \times 1$ influence vector containing the degree of freedom where the actuator output $f$ is acting. In modal coordinates, the displacement vector is expressed as $X = \Phi Z$, where $Z$ represents the modal amplitude vector. Assuming proportional damping and premultiplying Equ. 3.2 by $\Phi^T$ and substituting for $X$, Equ. 3.2 transforms into,

$$\Phi^T M \Phi \ddot{Z} + \Phi^T C \Phi \dot{Z} + \Phi^T K \Phi Z = \Phi^T b_1 f$$  \hspace{1cm} (3.3)\]

which gives,

$$\ddot{Z} + diag(2\xi_1\omega_1)\dot{Z} + diag(\omega_i^2)Z = diag(\mu_i^{-1})\Phi^T b_1 f$$  \hspace{1cm} (3.4)\]
where $\mu_i$, $\omega_i$ and $\xi$ are mass, frequency and the damping parameters, associated with the $i$-th mode. Assuming that the displacement measured at a certain point is given $x = b_2^TX$, where $b_2$ is the corresponding $m \times 1$ influence vector, the transfer function $G(s)$ between the input $f$ and the displacement $x$ is given by,

$$G(s) = b_2^T \Phi \text{diag} \left[ \frac{1}{\mu_i(s^2 + 2\xi_i \omega_i + \omega_i^2)} \right] \Phi^T b_1$$

$$= \sum_{i=1}^{n} \frac{\phi_i(x_a)\phi_i(x_s)}{\mu_i(s^2 + 2\xi_i \omega_i s + \omega_i^2)}$$  \hspace{1cm} (3.5)

where $\phi_i(x_a) = b_1^T \phi_i$ is the value of the $i$-th mode shape at the DOF associated with the actuator. Similarly, $\phi_i(x_s) = b_2^T \phi_i$ corresponds to the sensor location. $G(s)$ is the summation of the individual modal responses. The denominator of $G(s)$ gives the resonance frequencies or open loop poles of the structure and these are independent of the location of the sensor and the actuator. Since $\phi_i(x_a)$ and $\phi_i(x_s)$ are explicitly present in the numerator, the location of the zeros in the complex plane are dependent on the location of the sensor and the actuator. A collocated sensor and actuator configuration is always characterized by an alternating pole zero configuration of $G(s)$ in the complex plane, as discussed in Miu [18], Preumont [19] and Wie [81]. This pattern of pole-zero configuration does not change unless the sensor and the actuator are physically separated. Unconditional stability of the system in the closed loop is ensured since the closed loop poles converge to the zeros for increasing gain and are confined to the left side of the complex plane. The situation is totally different in case of non collocated control. It is shown in [18] that removing the sensor away from the actuator causes a migration of the zeros along the imaginary axis towards $\pm j\infty$ and thus the interlacing property of the poles and the zeros is no longer guaranteed. This affects the closed loop loci of the poles of the system, which may no longer be confined to the left half of the complex plane. Beyond a certain value of the separation between the sensor and actuator, a non collocated system is always non minimum phase (Flashner et al [82], [83]) and impossible to control. Cannon et al have experimentally shown difficulties in controlling flexible structures with a non collocated sensor and actuator configuration in [84]. The evolution of the zeros from a collocated sensor and actuator configuration to various degrees of separation of the sensor and actuator is demonstrated in Figure 3.3, for a simply supported undamped beam with length $l = 1$. It is observed that with increasing separation of the sensor and actuator, the system evolves from an alternate pole zero configuration to a pole zero cancelation situation, followed by a non alternating configuration of the poles and zeros with the eventual appearance of non minimum phase zeros on the positive real axis.
Figure 3.2: A basic vibration control system in a simply supported beam

Figure 3.3: Evolution of zeros with migration of the sensor with respect to the actuator
### 3.3.2 Various methods of active damping

Table 3.1, taken from Chapter 5 of the book of Preumont [19], summarizes the various kinds of active damping strategies with different kinds of collocated sensors and actuators. The first column represents different kinds of sensors and the first row represents different kinds of actuators.

<table>
<thead>
<tr>
<th>(gH(s))</th>
<th>Force</th>
<th>Strain ((d_{31}) piezo)</th>
<th>Linear ((d_{33}) piezo)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Displacement</td>
<td>(\frac{\text{Lead}}{s})</td>
<td>(\text{Direct Velocity Feedback})</td>
<td>(\text{Integral Force Feedback})</td>
</tr>
<tr>
<td></td>
<td>(g &gt; \frac{s}{s+a})</td>
<td>(g)</td>
<td>(\frac{-g}{s})</td>
</tr>
<tr>
<td>Velocity</td>
<td>(g/s)</td>
<td>Positive Position Feedback</td>
<td>(\frac{-g \omega_f^2}{s^2+2\xi_f \omega_f s+\omega_f^2})</td>
</tr>
<tr>
<td>Acceleration</td>
<td>(g)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(\frac{g}{s^2+2\xi_f \omega_f s+\omega_f^2})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Strain ((d_{31}) piezo)</td>
<td>(\frac{-g \omega_f^2}{s^2+2\xi_f \omega_f s+\omega_f^2})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Force</td>
<td>(\text{Integral Force Feedback})</td>
<td>(\frac{-g}{s})</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.1: Collocated active damping using various kinds of sensors and actuators

Work done by various researchers on active damping with acceleration feedback with second order controllers can be found in Sim et al [85], Loix et al [86], Goh et al [87] and Hanagud et al [88]. Use of second order filters as virtual passive vibration absorbers is demonstrated in Juang et al [89] and Bruner et al [90]. The positive position feedback strategy (PPF) was proposed by Caughey et al [91] and [92] for piezoelectric sensors and actuators. The integral force feedback technique is applied for active damping of trusses by Preumont et al [93].

Active damping with velocity feedback is considered in the present study for stabilization of machine tool chatter. Turning and milling operations involve uncertainties due to changes in the dynamics of the tool and workpiece system during the cutting process.
Active damping does not require a model of the system and is robust if a collocated sensor and actuator configuration is used. This makes it a potential candidate for chatter control in machining operations. Both milling and turning operations are associated with large amplitude vibrations and the actuator is required to be immensely strong to have a control effect on the system. Thus hydraulic or inertial actuators are more suitable for application of active damping in this case. In the present study, active damping with proof mass dampers (also called Active Mass Damper (AMD)), using velocity feedback is considered. Since acceleration signals are easy to measure in comparison to velocity or displacement, active damping can be implemented by integrating the acceleration signal (using \( H(s) = \frac{1}{s} \)) and feeding it to the actuator. Equ. 3.4 for the SISO system is written for a collocated sensor-actuator configuration \((b_1 = b_2 = b)\) with an extra negative viscous force term, coming from the control, on the right hand side.

\[
\ddot{Z} + \text{diag}(2\zeta \omega_i)\dot{Z} + \text{diag}(\omega_i^2)Z = \text{diag}(\mu_i^{-1})(\Phi^T b f - g\Phi^T bb^T \Phi \dot{Z})
\] (3.6)

where \( g > 0 \) is the feedback gain. The closed loop transfer function \( G_c(s) \) between the force and the displacement is given in Equ. 3.7.

\[
G_c(s) = b^T \Phi \text{diag} \left[ \frac{1}{\mu_i(s^2 + (2\zeta \omega_i + g\frac{\phi_{bi}^2}{\mu_i})s + \omega_i^2)} \right] \Phi^T b
\]

\[
= \sum_{i=1}^{n} \frac{\phi_{bi}^2}{\mu_i(s^2 + (2\zeta \omega_i + g\frac{\phi_{bi}^2}{\mu_i})s + \omega_i^2)}
\] (3.7)

The introduction of velocity feedback reduces the modal response, due to an increase in the damping coefficient in the denominator in Equ. 3.7. The response is further reduced with increase in the feedback gain \( g \). Due to a collocated sensor and actuator configuration \( g\frac{\phi_{bi}^2}{\mu_i} \) is always positive, which augments of damping, resulting in unconditional stability in the closed loop. The equation also shows that the efficiency of the control depends on where the sensor and the actuator are located. As shown by Aubrun [94], the sensor and the actuator need to be placed where \( \phi_{bi} \) is large, implying good observability of the mode. If \( \phi_{bi} = 0 \), meaning that the mode is not observable, the mode cannot be controlled or damped. The distribution of strain energy is proposed as a compound index for checking the observability of various modes for active damping with strain based sensors and actuators in Preumont et al [93]. In case of inertial actuators and accelerometer sensor, the sensor and the actuator should be placed at a position of maximum displacement or velocity. The corresponding location for a cantilever beam is at the beam tip, where all the modes of the structure are observable and therefore controllable.

### 3.3.3 Active damping with AMD

The AMD is an electromagnetic system with a conducting coil and a magnet. Due to a current input into the coil, there is a relative motion between the coil and the magnet. If
the coil is fixed, the magnet acts as a moving mass and generates inertial forces on the supporting structure. The magnet may be fixed in some cases and a mass can be attached to the moving coil. A prototype AMD is described in Zimmerman et al in [95].

Figure 3.4 shows the conceptual idea behind active damping of a simply supported beam with an AMD, collocated with an accelerometer. The spring of the AMD may be mechanical or electromagnetic. The dashpot represents the electromagnetic damping due to the Lorentz force, which arises due to the relative velocity between the magnet and the current conducting coil. Active damping by velocity feedback is implemented by sensing the absolute acceleration of the beam and feeding it through an integrator to the actuator. The control setup is not an exact collocated system, since the sensed signal is absolute acceleration of the beam and the control force is a relative force between the actuator and the structure. However, this is the closest approximation to collocated feedback control, achievable in the present setup.

The effect of actuator dynamics is now discussed. Let \( i \) be the current input into the actuator, leading to an electromagnetic force \( f_c \), which acts in opposite directions on the beam and actuator system. The dynamics of the motion of the beam and the coupled actuator is as follows.

\[
\begin{align*}
M_a \ddot{X}_a + C_a (\dot{X}_a - b^T \dot{X}_s) + K_a (X_a - b^T X_s) &= f_c \\
M_s \ddot{X}_s + C_s \dot{X}_s + K_s X &= -bM_a \ddot{X}_a - bf_c \\
&= bC_a (\dot{X}_a - b^T \dot{X}_s) + bK_a (X_a - b^T X_s) - bf_c 
\end{align*}
\]

(3.8)

\( M, C, K \) and \( X \) represent the mass, damping, stiffness matrices and the displacement vector of the beam-actuator system with the subscript \( s \) and \( a \), referring to the beam.
and mass damper respectively. $b$ represents the influence vector, containing the degree of freedom, where the active mass damper and the beam are coupled to each other. Following Matunaga et al [96], the beam equation is written in modal coordinates and the combined system in Equ. 3.8 is rewritten in Equ. 3.9. The equation shows that the actuator and the beam system are coupled and therefore, the dynamics of the actuator cannot be neglected if an active control scheme is implemented on the system.

$$ \begin{bmatrix} I & 0 \\ 0 & M_a \end{bmatrix} \begin{bmatrix} \hat{Z}_s \\ \hat{X}_a \end{bmatrix} + \begin{bmatrix} \text{diag}(2\xi_{\omega_i}) + \text{diag}(\frac{1}{\mu_i})\phi^TbC_a b^T\phi & -\text{diag}(\frac{1}{\mu_i})\phi^TbC_a \\ -C_a b^T\phi & -C_a b^T\phi \end{bmatrix} \begin{bmatrix} \hat{Z}_s \\ \hat{X}_a \end{bmatrix} + \begin{bmatrix} \text{diag}(\omega_i^2) + \text{diag}(\frac{1}{\mu_i})\phi^TbK_a b^T\phi & -\text{diag}(\frac{1}{\mu_i})\phi^TbK_a \\ -K_a b^T\phi & -K_a b^T\phi \end{bmatrix} \begin{bmatrix} Z_s \\ X_a \end{bmatrix} = \begin{bmatrix} -\phi^Tb_{fc} \\ f_{fc} \end{bmatrix} $$ (3.9)

The system is stable since the equivalent mass, damping and stiffness matrices are positive definite. The previous sections do not consider the effect of actuator dynamics and a colocated sensor and actuator configuration ensures unconditional stability in the closed loop (except PPF which is not unconditionally stable). However, there is always a destabilizing effect of actuator dynamics in the control of flexible structures, due to the finite actuator dynamics [91]. The PPF strategy, which is more suitable for a piezoelectric sensor and actuator, makes the closed loop system immune to actuator dynamics, within a finite value of the feedback gain [91]. [92].

The choice of the resonance frequency of the AMD is important for the stability of the closed loop system. The effect of actuator frequency on active damping of the simply supported beam with velocity feedback is presented in Figure 3.5. Figure 3.5 a) shows the case, where the actuator frequency is below the modal frequencies of the structure. The Root Locus plot shows that the system is not unconditionally stable. However, the structural modes get considerably damped until the actuator mode becomes unstable. In Figure 3.5 b), the actuator mode is between the first and the second bending modes. This has a destabilizing effect on the first mode. The dynamics of the active mass damper is similar to a highly damped second order high pass filter. The bode plot between the current input and the force output of the actuator starts with a 40 dB rise and becomes flat for all frequencies beyond the resonant frequency. The initial phase of the transfer function is 180 degrees, which goes through 90 degrees at the resonance frequency of the actuator to eventually become 0 degree at higher frequencies. Therefore, for frequencies below the actuator frequency, the phase of the feedback signal is changed, which may lead to instability of all modes below the actuator mode. Thus, in order to apply active damping with an inertial actuator, the control loop is most efficient when the frequency of the actuator is lower than the lowest modal frequency of the structure [97]. Garcia et al [98] have experimentally demonstrated the relative effectiveness of velocity feedback with AMDs over tuned mass dampers, even though augmentation of damping occurs via both techniques. The following sections investigate the effect of active damping on the system stability in turning and milling.
Figure 3.5: Root locus plot of simply supported beam with velocity feedback with actuator mode a) less than the first modal frequency and b) between first and second structural modes

3.4 Active damping of chatter in turning

3.4.1 Physical explanation

Active damping strategies have not been explored as much as online speed control techniques for suppression of chatter. Recently, collocated active damping of chatter vibration in thin flexible workpieces has been applied by Sims et al [99], using piezoelectric sensors and actuators. Active damping with velocity feedback has been implemented in the works of Cowley et al [100] and Tewani et al [101]. Recently, active damping using integral force feedback strategy is implemented in a smart tool holder by Harms et al [102]. Referring back to Chapter 1, if the effect of velocity feedback with gain $g$ is included, the stability limit as expressed in Equ. 1.16, is modified into,

$$K_{lim} = \left( c + g \right) \frac{\omega_c}{\sin \omega_c T} \quad (3.10)$$

Since the damping coefficient is modified, a general rise in the stability limits is expected. The example of a SISO system, with a natural frequency of 47 Hz and damping ratio $\xi$ of 1% is presented to demonstrate the effect of active damping. The stability lobes are plotted for two arbitrary values of feedback gains of 250 and 500 units in Figure 3.6 a). The diagram shows not only a rise in the stability limits but also a slight shift of the lobes towards the right, under the influence of active damping. This shift can be explained from Figure 3.6 b). It is observed that the chatter frequency for a chosen spindle speed can change under the effect of active damping. Thus, referring to Equ. 3.10, the limiting $K_{cut}$
will be affected not only by a change in the damping due to the feedback, but also due to a change in term $\omega_c/\sin \omega_c T$, due to modified chatter frequencies. An obvious question arises as to by what factor would the stability limits be enhanced for various spindle speeds by the application of active damping. This is demonstrated in Figure 3.7 a), where the ratio between the limiting cutting stiffness for two values of feedback gains and the corresponding value without any active damping is demonstrated. It is found that the enhancement of stability limit is the highest for those spindle speeds, where the stability limit is low, such as 3790 RPM. For spindle speeds of 2700, 2840 and 8000 RPM, with originally high stability limits, the factor of enhancement is not impressive. This can be explained again by referring to Fig. 3.7 b), which is a plot of $\omega_c/\sin \omega_c T$ for the different values of the active damping feedback gain. It is observed that for the high stability regions, there is a reduction in the absolute value of the quantity. This counteracts the effect of increased damping due to the feedback gain $g$ in Equ. 3.10 and therefore affects the factor of increase in stability. In low stability regions (e.g. 3790 RPM), the change in the value of $\omega_c/\sin \omega_c T$ is negligible and the stability limit directly depends on the damping coefficient. Therefore, the factor of increase in the low spindle speed cases is much higher in comparison to that in the high stability regions of the stability lobe diagram. This explains the non-uniform relative enhancements due to active damping for different spindle speeds.
Figure 3.6: a) Comparison of stability lobes with and without active damping b) Chatter frequencies with and without active damping
From this discussion, an advantage can be seen in using active damping of chatter over spindle speed change technique [58]. Machining involves uncertainties in the system parameters, due to changes in the configuration of the machine tool structure. This causes changes in the system dynamics and the stability lobes. As an example, Figure 3.8 shows the effect of changes in the stiffness of the system on the stability lobe diagram. It is found that the high stability region, where the spindle speed frequency is equal to the natural
frequency of the system, shifts due to changes in the stiffness values. The spindle speed selection strategy may face problems in such an unpredictable situation. The control technique no doubt allows one to machine with high depths of cut at the high stability limit spindle speeds. Changes in the system dynamics may shift the stability lobes horizontally, leading to chatter in a previously stable situation. There is no such problem while machining at the regions of low stability, since a shift of the stability lobe diagram does not reduce the stability limit. Active damping causes the best enhancements of stability in these regions and therefore offers a wider area of stable machining with less uncertainty.

![Changes in the stability lobe diagram due to changes in the stiffness of the machine tool structure](image)

Figure 3.8: Changes in the stability lobe diagram due to changes in the stiffness of the machine tool structure

### 3.4.2 Root Locus demonstration of active damping effect

In this subsection, Root Locus plots of the closed loop eigenvalues for the chosen spindle speeds under active damping are demonstrated. Arbitrary feedback gains of 250 and 500 units are considered. The general effect of active damping is to shift the structural pole towards the left, due to enhanced damping. This implies that a higher cutting stiffness is necessary to make the system unstable, causing a rise in the stability limit. There is an increase in chatter frequency, due to the application of active damping. For 8000 and 3790 RPM in Figures 3.9 a) and b), the structural pole crosses the imaginary axis at higher frequencies than the chatter frequency without active damping. In Figure 3.9 c), the situation is close to a pole zero cancellation since the zero due to the delay is very close to the structural pole. It is interesting to see that due to active damping, the structural pole no longer becomes unstable. The delay pole triggers the instability. Therefore active damping
Figure 3.9: Effect of active damping for a) 8000 RPM b) 3790 RPM c) 2840 RPM d) 2700 RPM

not only causes additional damping, but may also change the nature of the instability. For 2700 RPM in Fig. 3.9 d), the delay pole becomes unstable with or without active damping and the structural pole remains on the left side of the complex plane.

Certain observations can be made from the Root Locus plots. The structural poles are affected more by the application of active damping than the delay poles. The delay poles, due to distant location from the imaginary axis do not undergo any appreciable change in damping. For the low stability regions like 3790 RPM, active damping is relatively more effective in enhancing the stability limit than in the high stability regions, where the delay pole may become unstable. Figures 3.9 a) and b) show two cases, where the structural pole becomes unstable. In the former case (8000 RPM), the stability limit is originally high in comparison to 3790 RPM. Due to active damping, the loci of the closed loop structural poles are lengthened in both cases, indicating a rise in the stability limit. However, for 3790 RPM, the relative increase is more pronounced. This explains the effectiveness of active damping for low stability spindle speeds.
3.5 Active damping of chatter in milling

3.5.1 Observations

The reasons behind the choice of active damping, as a chatter control strategy in milling, are as follows. The developments in Chapter 2 have shown the complexities involved in milling chatter, due to the time varying nature of the governing equation, different mechanisms causing instability and also dependence on the kind of milling operation, changes in system dynamics etc. Involvement of these complexities require the chatter control strategy to be model independent and robust to all kinds of uncertainties. Easy implementation is also another issue. Active damping with a collocated sensor actuator configuration has advantages of robustness and may be suitable for implementation in a machining environment. In this section, numerical time domain simulations are performed to investigate the effect of active damping with velocity feedback on the stability of various milling operations. The tool model, described in Table 2.1 is taken for the simulation. Since the system has flexibilities in the two orthogonal directions, actively damping each direction individually and both directions simultaneously are investigated. Feedback gains are applied in the two directions in order to increase the closed loop damping values to 5% from the original 1% in both directions. Figure 3.10 a) shows the effect of actively damping the X direction, keeping the Y direction unaltered. It is observed that the stability limits are mostly enhanced at those spindle speeds, associated with Hopf bifurcation of the mode in the X direction. Figure 3.10 b) shows the case of the structure with the Y direction, actively damped. The number of occurrences of Hopf bifurcation from the Y direction is moderate except for the high spindle speed region. The effect of enhancement of stability is prominent at those spindle speeds. In both cases, the Flip bifurcation regions do not experience a substantial rise in the stability limits. Active damping in both directions, as shown in Figure 3.11 a), enhances stability limits at all spindle speeds in upmilling, where the instability is Hopf bifurcation, irrespective of the direction from which it may be triggered. The factor of enhancement in the stability limits is shown as a ratio of limiting $K_{cut}$ with and without damping in Figure 3.11 b). It is found that active damping is more efficient in the Hopf Bifurcation regions, in comparison to the Flip Bifurcation spindle speeds. The former constitutes some parts of the low stability limit regions of the stability lobe diagram. The same observation is made for downmilling, as shown in Figure 3.12 a) and b). Since chatter in milling is associated with different mechanisms of instability, it makes more sense to damp both directions simultaneously in order to achieve a good enhancement of stability limits for a wide range of spindle speeds.

3.5.2 Physical explanation

The studies show that application of active damping in milling results in an unequal enhancement in the stability limit levels for different spindle speeds. Hopf Bifurcation regions are most enhanced due to active damping. The Flip Bifurcation regions are not affected much under the influence of active damping. The physics behind this can be explained as
follows. Hopf bifurcation is generally associated with the instability of a structural mode. Active damping enhances modal damping and thereby automatically stabilizes the mode against chatter. In case of Flip bifurcation, the system at instability oscillates with a basic chatter frequency equal to one half of the tooth passing frequency. This may not be necessarily close to a modal frequency of the system. Therefore, application of active damping would not enhance the stability at those spindle speeds.

Figure 3.10: Effect of enhanced damping in a) X direction and b) Y direction on stability limits for upmilling
Figure 3.11: a) Effect of enhanced damping in both directions on stability limits for up-milling and b) Factor of enhancement
Figure 3.12: a) Effect of enhanced damping in both directions on stability limits for down-milling b) Factor of enhancement

However, it is observed in Figure 3.10 b), application of active damping in the Y direction is enhancing the stability limit to a certain extent around 900 RPM, which is a Flip bifurcation region. This can be explained as follows. Flip bifurcation around 900 RPM is associated with tooth passing frequency at 60 Hz and a basic chatter frequency at 30 Hz, which is equal to the modal frequency in the Y direction. Therefore, the spectra of the displacement would contain a strong 30 Hz component. Damping the Y direction
would definitely stabilize the system and an enhancement of stability limit is expected. Active damping of $X$ direction does not change the stability limit at this spindle speed, as seen in Figure 3.10 a). This is due to the fact that there is no effect on the 30 Hz mode due to enhanced damping in the $X$ direction.

### 3.6 Conclusion

This chapter presented a survey of various strategies of chatter stabilization and discussed about active damping with collocated sensor and actuator configuration. Among active damping strategies, velocity feedback is chosen for practical implementation in the current work, which would be described in chapters 4 and 5. The choice of an active mass damper as the actuator is attributed to the need of a powerful force actuator, capable of applying damping forces to large milling machine systems. Numerical simulations show that active damping is quite effective in raising the stability lobes in the low stability regions for both turning and milling. This may be seen as an advantage over chatter stabilization using spindle speed change technique, since the low stability regions are far less sensitive to uncertainties in the system parameters in comparison to the highly stable parts of the stability lobe diagram. The studies also show that in case of a MDOF system, it makes more sense to damp all flexible directions in order to have a good enhancement in the stability limits.
Chapter 4

Demonstrator for chatter in turning

4.1 Introduction

The previous chapters have dealt with numerical simulations on chatter instability. This chapter is devoted to the development of a ”Hardware in the Loop” demonstrator for turning, constructed in the Active Structures Laboratory in ULB. The main motivation of the experimental work is to implement active damping as a chatter stabilization strategy. A good understanding of the chatter phenomenon is essential for this purpose. Experimental characterization of chatter is difficult in real machining conditions, due to the involvement of numerous parameters and the requirement of numerous cutting tests. The characterization process requires the system to be actually unstable and this may be detrimental to the life of the machine tool. The regenerative feedback model of chatter is however, quite well-established. Recent advances in signal processing technologies provide an alternative way to study chatter via a mechatronic simulator, without conducting actual cutting tests. The demonstrator provides an opportunity to investigate various aspects of chatter instability experimentally. It also includes an active damping setup to experimentally study the effectiveness of the control strategy in chatter suppression.

4.2 The demonstrator

4.2.1 Setup description

Figure 1.5 shows that chatter is due the instability in the interaction between two independent entities; the MDOF structural dynamics and the cutting process. This feedback model provides the conceptual idea of the mechatronic demonstrator for chatter. An aluminium cantilever beam is used to represent the MDOF dynamics of a turning machine and a voice coil actuator at its end generates the cutting force signal. A corner cube reflector is mounted on the other side of the tip, which is a part of a HP laser interferometer setup (description in HP manual [103]), acting as the displacement sensor. The 16 bit position information from the interferometer is passed on to a DSP board where the regenerative
cutting process is simulated in real time. The cutting force values are calculated and passed through the digital to analog converter of the DSP board to a current amplifier and finally into the voice coil actuator. The closed loop system thus consists of the beam, which represents the machine tool structure, the sensor and actuator, all of which represent the hardware component and a software layer simulating the regenerative cutting forces. A photograph of the setup is shown in Figure 4.1. The "Hardware in the Loop" concept is illustrated in Figure 4.2.

![The demonstrator setup](image)

**Figure 4.1: The demonstrator setup**

The dSpace Control Desk software provides a graphic user interface, enabling the change of $K_{cut}$ and $T$ in real time on the DSP board. In other words the spindle speed and the cutting condition can be changed just as in real machining. Certain combinations of the two parameters, lead to an unstable feedback loop, resulting in a growth in the oscillations of the beam, thus representing a chatter situation. The feasibility of chatter control by active damping is also investigated in the latter portion of the study. The beam has an inertial actuator, also called an AMD (manufactured by Micromega Dynamics), mounted on its side, (details in Figure 4.5) to actively damp the system.
4.2.2 Numerical simulations

This portion deals with investigations on stability with a numerical model of the beam structure. The state space model of the beam is generated, using the Matlab based Structural Dynamics Toolbox, developed by SDTools [104], by curve fitting on the experimental frequency response plot, using the Pole-Residue method in frequency domain. The identification is done without dismounting the AMD, in order to include its dynamics and generate a realistic model for the system. The properties of the system are tabulated in Table 4.1.
<table>
<thead>
<tr>
<th>Mode</th>
<th>Frequency (Hz)</th>
<th>Damping (%)</th>
</tr>
</thead>
<tbody>
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<td>Actuator</td>
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<td>19.6</td>
</tr>
<tr>
<td>Mode 1</td>
<td>46.8</td>
<td>1.9</td>
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<td>Mode 2</td>
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<td>1</td>
</tr>
<tr>
<td>Mode 3</td>
<td>62.2</td>
<td>0.8</td>
</tr>
</tbody>
</table>

Table 4.1: Modal frequencies and damping properties for the demonstrator setup

The Root Locus Method is used to generate the stability lobe diagram and to check for the nature of instability at various spindle speeds. The stability lobe and the chatter frequency diagram for the demonstrator are plotted in Figure 4.3. As in the case of the SDOF system, there are different sources of instability, as shown in the figure. The interaction between the structural poles and the delay is similar to that of the SDOF system, the only difference being the involvement of multiple modes in the process of instability.

4.2.3 Experimental simulation of chatter

Stability lobe diagrams are also generated experimentally. A value of the delay $T$ is chosen. $K_{cut}$ is increased step by step and for each step the displacement response of the beam, due to a computer generated impulse excitation is checked on the computer display. A stable system is characterized by a decaying response in contrary to an unstable response, which grows with time. In case the system is critically stable, the response to impulse does not grow or decay with time and the oscillations are sustained. The corresponding value of $K_{cut}$ is stored as the limiting value for the chosen spindle speed. The chatter frequency is measured from the displacement signal in a FFT analyzer. This experiment is repeated for many spindle speeds. The experimental stability lobes and chatter frequencies are compared with those obtained by the Root Locus Method in Figure 4.4. Three sets of experiments are performed in order to check for the repeatability of the results. The match between experimental and the numerical analysis is good.
Figure 4.3: Stability lobes and chatter frequency diagrams for the demonstrator
4.3 Application of active damping

Active damping by velocity feedback is adopted as the chatter control strategy, as discussed in the previous chapter. The demonstrator for turning consists of an electromagnetic AMD, which is basically a spring-mass-dashpot system coupled to a voice coil actuator. This generates inertial forces which act upon the beam structure. This adds an extra layer to
the "Hardware in the Loop" system. Two loops exist in the setup, i.e., the regeneration-cutting process loop and the active damping loop. This is illustrated in Figure 4.5. An accelerometer collocated with the AMD senses the acceleration signal, which is integrated and multiplied by a gain $g$ through a controller block and fed into the coil of the AMD. The AMD thus introduces damping into the structure. The effect of active damping on the system under chattering conditions is now investigated.

A Root Locus plot is generated for increasing $K_{cut}$ at a spindle speed of 2000 RPM, without any active damping and for arbitrary feedback gains of 5 and 10 units. The identified model of the beam is used in this numerical exercise. In all the three cases, the instability arises from the first mode as shown in Figure 4.6. The system at 2000 RPM, without active damping, has an initial damping of 1.9% for the first mode, 1% for the second mode and 0.8% for the third mode. With active damping the first mode is heavily damped to 5% and 9% for 5 and 10 units of feedback gain respectively but the effect on the other two modes is not much. The loci of the eigenvalues for the first mode are much longer than that without any active damping. This explains the rise in the stability of the system against chatter. The stability lobe diagrams with and without active damping are compared in Figure 4.7. The effects are similar to the case of the SDOF system. It is also observed that active damping changes the source of instability at certain spindle speeds.

Figure 4.5: The chatter demonstrator with active damping layer
Figure 4.6: Root locus of eigenvalues for increasing $K_{cut}$ without active damping and with feedback gains $g = 5$ and 10 units.

Figure 4.7: Effect of active damping on stability lobe diagram (Numerical)
The enhancement of stability due to active damping is found to be non-uniform at various spindle speeds. The enhancement is more impressive in the low stability region of the stability lobe diagram (e.g. 3500 RPM in Figure 4.7). In the originally high stability region (e.g. between 2500 and 3000 RPM) the stability enhancement is not very impressive. This proves that active damping is most effective in the low stability limit areas of the stability lobe diagram, where the structural pole is becoming unstable. Highly stable regions, where delay pole contribute to instability are enhanced to a lesser extent due to active damping.

Finally the stability lobes are determined experimentally to demonstrate the effect of active damping. As described earlier on experimental simulation of chatter, the demonstrator is set to chatter at several spindle speeds with the active damping feedback loop working. The results are plotted in Figure 4.8. Feedback gains of 5 and 10 units are chosen for the experiment. A rise in the level of stability limits is observed, confirming the role of active damping in increasing the stability against chatter.

![Figure 4.8: Effect of active damping on stability lobe diagram (Experimental)](image)

4.4 Conclusion

This chapter discusses about the development of a Hardware in the Loop demonstrator for turning, which provides an alternative way of investigating the instability in a laboratory environment, without conducting actual cutting tests. The numerical simulation and experimental stability analysis results match well, confirming that the demonstrator is able
to simulate chatter realistically. An active damping setup is implemented on the system as an independent layer along with the chatter simulation layer. It is observed that the effect of active damping is more pronounced in the low stability areas of the stability lobe diagram.
Chapter 5

Mechatronic simulator for chatter in milling

5.1 The Hardware in the Loop setup

5.1.1 Description of setup

In milling, the metal is removed from the workpiece by a combination of forward motion and rotation of the cutting tool. Under the effect of periodic cutting forces, the flexible tool-workpiece system undergoes displacements. This along with the rotation of the tool creates the periodic regenerative effect and generates milling forces which further excite the machine tool structure. The closed loop milling model is shown in Figure 5.1.

The conceptual model of a 2 DOF demonstrator for milling is shown in Figure 5.2. The multi degree of freedom (MDOF) dynamics of the flexible machine tool and workpiece system is simulated by a rectangular hollow cantilever beam. Two voice coil actuators are attached to the free end of the beam in order to generate excitations in the two orthogonal directions. This simulates the cutting forces, acting on the cutting tool. Two different displacement sensor setups have been implemented on the demonstrator. In the initial stages of the work, the displacements in the two directions were sensed by eddy current sensors, manufactured by KAMAN Instrumentation. Later, the eddy current sensors were replaced by a ZYGO laser interferometer setup and the associated structural changes are discussed in a later section of this chapter. The sensed displacement signals are fed into a DSP board, where a time domain simulation of the cutting forces is running. The dSpace ControlDesk software provides a Graphical User Interface (GUI) to change the various cutting parameters, such as the axial width of cut, the spindle speed, the feed, the angles of entry and exit. The software calculates the system of cutting forces according to the various cutting conditions, specified by the user. The calculated force signals are directed to current amplifiers through the output resources of the DSP board. The current amplifiers generate the necessary current to drive the voice coil actuators, according to the calculated cutting forces. For a chosen spindle speed and cutting condition, a chatter situation can be triggered by gradual increase in the value of \( K_{\text{cut}} \), until the oscillations of the beam start to
grow. The cutting force or the displacement signals can be analyzed in a Fourier analyzer and the frequency content of the signal under stable and unstable conditions can be studied and conclusions can be made as to whether the instability is a Hopf or a Flip Bifurcation. This helps in characterization of the chatter instability in milling. The demonstrator also has an active damping setup, consisting of a couple of proof mass dampers also called AMDs, collocated with accelerometers, located at an intermediate distance on the beam (Refer to Figure 5.12). Details of the control setup will be dealt with in a section devoted to active damping of chatter.

Figure 5.1: The dynamic milling model
5.1.2 Time domain simulations

Time domain simulations are performed to demonstrate the various aspects of chatter instability. For this purpose, a state space model of the mechanical setup is identified, using the MATLAB based Structural Dynamics Toolbox, developed by SDTools [104]. The identified frequency bandwidth is 100 Hz. The identified modal parameters are presented in the following table.

<table>
<thead>
<tr>
<th>Modes</th>
<th>Frequency (Hz)</th>
<th>Damping (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Active Mass Damper X direction</td>
<td>8</td>
<td>20</td>
</tr>
<tr>
<td>Active Mass Damper Y direction</td>
<td>8.9</td>
<td>14</td>
</tr>
<tr>
<td>1st Flexible Mode X direction</td>
<td>37</td>
<td>2</td>
</tr>
<tr>
<td>2nd Flexible Mode X direction</td>
<td>66.9</td>
<td>1</td>
</tr>
<tr>
<td>1st Flexible Mode Y direction</td>
<td>60.6</td>
<td>1.1</td>
</tr>
<tr>
<td>2nd Flexible Mode Y direction</td>
<td>72.8</td>
<td>0.9</td>
</tr>
</tbody>
</table>

Table 5.1: Modal frequencies and damping properties of the beam-PMD setup
Unstable and stable situations are compared for a spindle speed of 1500 RPM and varying cutting conditions. The PSD of the displacement signals (obtained from the time domain simulations) for unstable situations are shown by continuous lines and those corresponding to stable situations are shown by dotted lines. In Figure 5.3, a chatter situation for slotting with the 4 teeth tool is investigated for 1500 RPM. The PSD plot of the displacements shows only a single frequency and no tooth passing frequency. Due to the 90 degree separation between successive teeth and two teeth always being in contact with the workpiece, there is no periodicity in the feed forces and the regenerative process. Therefore, the tooth passing frequencies are absent. The instability is similar to the case of turning since there is only one chatter frequency. The same chatter frequency arises in the two directions, since there is a coupling between the two directions through the milling coefficient matrix $\Psi$. There may also be some mechanical coupling between the two directions since the placement of the actuators may not be exactly on the vertical middle line of the beam. So if there is some vibration at a particular frequency in one direction, it excites the other direction as well at the same frequency. In Figure 5.4, a chatter condition for

50% upmilling at 1500 RPM with 4 teeth is investigated. The tooth passing frequency at multiples of 100 Hz are marked by circles. The case is that of a Flip Bifurcation since the chatter frequencies (marked by black squares) exist at multiples of one half of the tooth passing frequency.

Figure 5.3: PSD of displacements for slotting at 1500 RPM (numerical)
Figure 5.4: PSD of displacements for 50% immersion upmilling at 1500 RPM (numerical)
For 50% immersion downmilling for the same spindle speed, the case is that of a Hopf Bifurcation, as shown in Figure 5.5. The dominant chatter frequency arises from the second flexible mode in the Y direction at 72 Hz. A low immersion upmilling example, with an angle of cut equal to 30 degrees, is examined in Figure 5.6. The instability is Flip Bifurcation since the chatter frequencies are at integer multiples of one half of the tooth passing frequency.

Figure 5.5: PSD of displacements for 50% immersion downmilling at 1500 RPM (numerical)
A popular way of identifying whether an instability is Hopf or Flip Bifurcation is by sampling the displacement signal, once per tooth passing period, as proposed by Schmitz [105]. The method is applied for chatter recognition by sensing the sound signal, generated in the cutting process. If during a Flip Bifurcation, the basic chatter frequency is 50 Hz and the tooth passing frequency is 100 Hz, a one per tooth pass sampling will generate 2 samples of displacement, during one period of oscillation of the structure. At the stability limit, assuming that the displacement neither grows or decays and the system oscillates with a single dominant frequency, the sampled signal value will always have two exact values. This is shown in Figure 5.7 a) for the low immersion milling example at 1500 RPM. This characteristic of the sample displacement signal helps to identify a Flip Bifurcation situation. In case of Hopf Bifurcation, there is no relationship between the basic chatter frequency and the tooth passing frequency. So sampling once per tooth passing period will not yield any particular distribution of the data with time, as in the case of Flip Bifurcation. A Hopf Bifurcation case for 50% immersion downmilling is shown in Figure 5.7 b).
Figure 5.7: Demonstration of once per revolution sampled data for a) Flip Bifurcation b) Hopf Bifurcation
5.1.3 Stability Limits

Stability limits, obtained by time domain simulations are compared with the results obtained by the Root Locus Method, using Padé Approximation for the delays (details in chapters 1 and 2) and replacing the periodic matrix $\Psi$ by the constant matrix $\Psi_0$. Numerical simulations are performed for 4 cases of milling operations with a 4 teeth tool: slotting, 50% immersion up and downmilling and a case of low immersion upmilling with angle of cut equal to 30 degrees. Figure 5.8, shows the results for slotting. The time domain and the Root Locus simulation results are exactly on top of each other. A high stability region is located between 400 and 600 RPM. The tooth passing frequencies in this region are close to 36 Hz, which is the first bending mode in the more flexible X direction.

![Graph showing stability limits comparison](image)

Figure 5.8: Comparison between stability limits obtained experimentally, by the Root Locus Method and by Time domain simulations for a slotting operation

In figures 5.9 a) and b), the stability limits corresponding to 50% upmilling and downmilling are plotted. For upmilling, the Root Locus method gives a good match at spindle speeds lower than 1000 RPM. Certain portions of the stability lobe diagram, where Hopf and Flip Bifurcations occur are identified by the oval shaped regions in the Figure 5.9 a). The stability limits, obtained by the Root Locus Method in the Flip Bifurcation regions do not match with their counterparts from the time domain simulation results.
Figure 5.9: Experimental and numerical stability limits for 50% immersion a) upmilling and b) downmilling

In Figure 5.9 b), the Root Locus results do not show the Flip Bifurcation regions. In
Figure 5.10, which is a study of a low immersion milling, the stability lobes obtained from time domain simulations show a prominent region, where Flip Bifurcation is occurring. This is not observed in the Root Locus result.

Figure 5.10: Experimental and numerical stability limits for low immersion milling

5.1.4 Experimental Results

Stability analysis is performed on the setup experimentally and the stability limits for different milling operations are plotted in Figures 5.8, 5.9 and 5.10 for comparison with numerical results. For a chosen spindle speed, $K_{cut}$ is increased stepwise until the onset of unstable oscillations, which are monitored on a PC display. The corresponding value of $K_{cut}$ is stored as the stability limit for the chosen spindle speed. In case of slotting, as shown in Figure 5.8, the experimental results follow the results from time domain simulation and the Root Locus technique. For upmilling, experimental results match very well with numerical results for spindle speeds less than 1400 RPM and deviate from the numerical results at high spindle speeds. In the case of downmilling, the experimental results follow the time domain simulation data but are not in good agreement with the results from the Root Locus method (which is normal since the Root Locus Method does not predict certain regions accurately). For low immersion upmilling in Figure 5.10, the experimental data match very well with time domain results. Figures 5.11 a) and b) represent the experimental PSD of the displacement in the X direction for a low immersion milling operation. A Hopf bifurcation scenario occurs for 750 RPM and a Flip bifurcation occurs for 1200 RPM.
Figure 5.11: Experimental demonstration of a) Hopf bifurcation (750 RPM) and b) Flip bifurcation (1200) RPM

The results of the numerical and experimental investigations can be summarized as follows. The experimental stability lobes follow the same trend as the time domain simulation results for all the milling examples. The match between the experimental and the
time domain results is good at low spindle speeds in comparison to higher spindle speeds. The reason can be attributed to finite bandwidth of the total setup. The performance of the closed loop system was not good beyond a tooth passing frequency of 100 Hz, which is equivalent to a spindle speed of 1500 RPM for a 4 toothed cutter. So the bandwidth of the demonstrator is 100 Hz.

The Root Locus Method has good agreement with the time domain and experimental results for high immersion milling. For partial immersion milling, there is agreement in the Hopf Bifurcation regions of the stability lobe diagram but the match is not good for spindle speeds where Flip Bifurcations occur.

## 5.2 Application of active damping for chatter control

A velocity feedback strategy for active damping, using a collocated sensor and actuator, is implemented in the milling demonstrator.

![Figure 5.12: Active damping setup for the 2 DOF demonstrator](image)

Certain structural modifications are made on the demonstrator setup. Two retroreflectors are mounted at the tip of the beam in the two directions as part of a ZYGO laser interferometry setup, used to measure the displacements in the two directions. The modal properties of the setup are thus changed (shown in Table 5.2). Active damping is applied in the two directions by two decentralized local acceleration feedback loops. The setup,
as shown in Figure 5.12, consists of two AMDs, collocated with a pair of accelerometers, which sense the acceleration signal in the two directions. These signals are fed into the controllers, shown by $H(s)$, integrated and multiplied by a gain and fed into the AMDs. The demonstrator now has two independent layers: the chatter simulation loop and the active damping loop. A photograph of the whole setup is shown in Figure 5.13.

![Figure 5.13: The experimental setup](image)
Two feedback gains are applied in each active damping loop and the structural properties with and without control are compared in Table 5.2. The case corresponding to "Controlled 2" has higher feedback gains in comparison to "Controlled 1" case. It is observed that higher feedback gains augment damping in both of the bending modes in the two directions. This is demonstrated by Root Locus plots in Figure 5.14 a) and b). It can be commented that the control setup is not an exact collocated system, since the feedback signal is the absolute acceleration of the beam at the sensor location and the control force is a relative electromagnetic force between the beam and the proof mass. However, the arrangement is the closest approximation of collocated control, achievable in the present setup. Thus, an alternate pole zero configuration of a collocated system is not observed in the Root Locus plots. In Figure 5.14 a), the actuator mode becomes unstable and the maximum damping ratio achievable in the first bending mode is about 50%. In the Y direction, the control system is able to apply critical damping to the system. However, experimentally such high damping ratios were not achievable. No particular reason could be found responsible for the low level of closed loop damping, since several factors such as spillover effect from the high frequency modes, finite bandwidth of the electronic hardware etc. may lead to an unstable closed loop. Thus the maximum achievable damping are 8.2% for X and 5.8% for the Y direction.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Uncontrolled</th>
<th>Controlled 1</th>
<th>Controlled 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ω (Hz) ξ (%)</td>
<td>ω (Hz) ξ (%)</td>
<td>ω (Hz) ξ (%)</td>
</tr>
<tr>
<td>Actuator X</td>
<td>9.1 14.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Actuator Y</td>
<td>7.5 18</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bending X</td>
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<td>25.1 6.7</td>
<td>24.3 8.2</td>
</tr>
<tr>
<td>Bending Y</td>
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<td>46.2 4.5</td>
<td>46.2 5.8</td>
</tr>
<tr>
<td>Torsion</td>
<td>67.6 0.8</td>
<td>67.8 0.8</td>
<td>67.6 0.5</td>
</tr>
</tbody>
</table>

Table 5.2: Modal frequencies and damping properties of the beam—AMD setup
Figure 5.14: Loci of closed loop poles due to active damping a) X direction b) Y direction

The effect of active damping on the stability of slotting, upmilling and downmilling is investigated numerically and experimentally. Figures 5.15 a) and 5.16 a) compare the numerical and the experimental data for the uncontrolled system and the two controlled cases for slotting and upmilling respectively. The experimental data follow the time domain simulation results reasonably. The experimental stability lobes are again compared in Figures 5.15 b) and 5.16 b). A general rise in the stability lobe diagrams is observed, especially in the low stability regions.
Figure 5.15: Comparison between a) numerical and experimental stability lobes b) experimental stability lobes for slotting with and without active damping
Figure 5.16: Comparison between a) numerical and experimental stability lobes b) experimental stability lobes for upmilling with and without active damping.

In case of downmilling, as shown in Figure 5.17 a), the match between the experimental and the numerical results is not very good. No reason could be put forward to explain the fall in the level of stability at certain spindle speeds for the higher value of feedback gain, as shown in Figure 5.17 b). However, the figure still shows that active damping is able to
raise the stability limits.

Therefore, active damping seems to be an efficient strategy for stabilizing chatter in milling, as in the case of turning. It is observed that active damping is more efficient in enhancing the stability in the low stability regions than the high ones. This can be seen as an advantage of active damping over spindle speed selection technique of chatter control [58]. The reason behind this statement is already explained in Chapter 4. The inherent complexities in milling also make active damping an attractive strategy for chatter control due to the technique’s robustness properties and ability of easy implementation.
Figure 5.17: Comparison between a) numerical and experimental stability lobes b) experimental stability lobes for downmilling with and without active damping
5.3 Conclusion

This chapter presents a 2 DOF mechatronic simulator for the study of chatter in the milling process. The demonstrator follows the regenerative milling process model and can experimentally simulate the occurrence of Hopf and Flip bifurcations in various kinds of milling operations. This shows the capability of the demonstrator to simulate regenerative chatter in milling realistically. The experimental stability lobes are found to follow the numerical time domain results quite well, except in the high spindle speed regions.

Active damping, as a chatter control strategy is also implemented in the setup. It is shown that active damping is able to stabilize all kinds of milling operations by raising the stability limits. The effect is more pronounced in the low stability regions of the stability lobe diagram of the uncontrolled system than the originally high stability regions. Thus, active damping can be proposed as a potential candidate for stabilization of chatter in real machining operations.
Chapter 6

General Conclusion

The main motivation behind the work in this thesis is to propose active damping as a potential chatter control strategy. The work presents a theoretical study to develop a background for chatter instability. This is followed by the practical development of two Hardware in the Loop demonstrators that allow experimental investigations on chatter and application of active damping for its stabilization.

To fulfil the theoretical objective, the regenerative chatter theory for turning and milling is followed in the present study. Spindle speed and structural damping are identified as the controlling parameters behind chatter. Spindle speed is shown to affect the phase difference between the inner and outer modulations of the chip thickness, i.e., the shape of the chip for various spindle speeds. The relationship between the stability and the shape of the chip is explained. Structural damping is found to have a proportional effect on the stability limit values for various spindle speeds. Enhancement of structural damping is found to stabilize the system against chatter, resulting in higher stability limits. This is the guiding principal behind the chatter control strategy, proposed in the present study. The Root Locus method is used to provide a control engineering perspective of chatter in turning. It is found that instability may be due to the structural pole or the delay pole. The low stability regions of the stability lobe diagram are found to be associated with the instability of a structural pole. Certain portions of the high stability regions are associated with the instability of the delay pole. Traditional stability analysis techniques identify that instability can only arise from a structural pole. In this study it is shown that the delay pole may also contribute to the instability.

Regenerative chatter in milling is more complicated than turning, since it is governed by a delay differential equation with periodic coefficients. The periodicity is equal to the tooth passing frequency. Frequency domain techniques cannot be directly used for stability analysis and most of the work in the present study on chatter in milling is performed via time domain simulations. Chatter in milling is characterized by the presence of multiple chatter frequencies. Two mechanisms, i.e., the Hopf and the Flip bifurcations are shown to cause chatter in milling. The former is generally associated with a basic chatter frequency, close to a structural mode. In case of Flip Bifurcation, the chatter frequency harmonics are located at odd multiples of one half of the tooth passing frequency. Hopf Bifurcation
is mostly associated with the low stability regions of the stability lobe diagram, whereas, Flip Bifurcation is associated with certain parts of the high stability regions.

Experimental studies on chatter are difficult in a real machining environment, due to associated uncertainties in the dynamics of the cutting process and the need for numerous cutting tests. This provides the motivation to construct demonstrators, which can simulate the chatter phenomenon in turning and milling in a laboratory environment and act as simple test beds to investigate active damping as a chatter control strategy. The demonstrators are built following the “Hardware in the Loop” concept. The hardware part consists of a cantilever beam, simulating the MDOF structural dynamics of the machine tool. The software part consists of a real time simulation of the cutting process (milling or turning), running on a DSP board. The closed loop interaction between the two entities leads to a chatter like situation without actual conduction of cutting tests and various aspects of the instability can be investigated. Both of the demonstrators can simulate chatter realistically. The demonstrator for milling is able to demonstrate Hopf and Flip Bifurcations at different spindle speeds.

Various chatter control strategies are summarized in the work as part of literature survey and collocated active damping by velocity feedback is chosen for numerical investigation and practical implementation on the demonstrators. The reason behind the adoption of this method is that active damping does not require a model of the system, is robust with a collocated sensor and actuator configuration and can be easily implemented. A general rise in the stability limits, due to active damping, is observed in the numerical study for both turning and milling. For both turning and milling, active damping is found to be more effective in the low stability regions in comparison to the high stability areas. This proves to be advantageous since enhancement of stability limits in the low stability areas generates a wider region of stable machining that is less vulnerable to changes in the system dynamics. It is also inferred from the study that since chatter in milling is associated with multiple mechanisms of chatter and many modes, it makes more sense to actively damp the identifiable flexible directions. In present study, this is achieved by actively damping the X and Y directions with two active mass dampers, via the velocity feedback strategy with a collocated sensor and actuator configuration. Experimental investigations show that stability lobes are raised due to active damping. This makes active damping a potential strategy for chatter control.

**Future plans**

The work undertaken in the present study is a part of the European Union sponsored SMARTOOL project, which intends to promote "smart" control technologies for chatter stabilization in machining processes. The work in the thesis demonstrates active damping as a potential chatter stabilization strategy. This approach is adopted in an industrial application, where an Active Mass Damper is being built by Micromega Dynamics S.A. for chatter control in a Soraluce Corporation manufactured milling machine. Improvements on the damper and industrial implementation are underway. Some improvements may be done
to the demonstrator. The bandwidth of the simulator may be increased to accommodate higher spindle speed simulations. If new control strategies are developed for chatter control in the future, they can be tested on the demonstrator.
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[67] P. Vanherck. Increasing milling machine productivity by use of cutters with non-


