MODELING OF PIEZOLAMINATED COMPOSITE SHELLS FOR VIBRATION CONTROL

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ABSTRACT

This paper develops the theory of piezolaminated shells. The fundamental equations governing the equivalent piezoelectric loads and sensor output are derived. The reciprocity between piezoactuation and piezosensing is pointed out. Piezoelectric shell finite elements are developed based on *Mindlin* elements. Different electrical boundary conditions are examined.

Key words: vibration control; piezoelectricity.

1. INTRODUCTION

The use of piezoelectric materials as actuators and sensors for noise and vibration control has been demonstrated extensively over the past few years (a review can be found in (Preumont, 1997)).

The design of control systems involving piezoelectric actuators and sensors requires an accurate knowledge of the transfer functions between the inputs and the outputs of the system. These are not easy to determine numerically, particularly for shell structures with embedded distributed actuators and sensors. The situation where they are nearly collocated is particularly critical, because the zeros of the transfer functions are dominated by local effects which can only be accounted for by finite elements. This motivates the present study.

The finite element modeling of plates and shells with embedded piezos has received a considerable interest in recent years. Stacking of quadrangular solid piezoelectric elements was considered in (Tzou and Tseng, 1990) and later in (Heyliger et al., 1996). A solid multilayered thin brick was proposed in (Ha et al., 1992). The analysis of piezoelectric shells as a layerwise assembly of solid piezoelectric triangular elements quadratic in-plane and linear through the thickness is proposed in (Tzou and Ye, 1996). These are tridimensional approaches and result in large problem sizes, requiring techniques such as the *Guyan* reduction to reduce the number of degrees of freedom. A multilayered composite piezoelectric *Kirchhoff* plate is derived in (Hwang and Park, 1993). To accomodate thick as well as thin shell structures, a 4-node bilinear *Mindlin* shell is proposed in (Suleman and Venkayya, 1995); a reduced integration of the stiffness matrix is used to avoid the transverse shear locking phenomenon for thin shells. The present study is also based on a *Mindlin* plate; however, the element used in the present study follows (Batoz and Dhatt, 1990) and does not require a reduced integration.

2. CONSTITUTIVE EQUATIONS

2.1. General

The constitutive equations of a linear piezoelectric material read (IEEE std, 1988).

$$\{T\} = [c^E]\{S\} - [e]^T\{E\} \quad (1)$$
$$\{D\} = [e]\{S\} + [\varepsilon^S]\{E\} \quad (2)$$

or alternatively

$$\{S\} = [s^E]\{T\} + [d]^T\{E\} \quad (3)$$
$$\{D\} = [d]\{T\} + [\varepsilon^T]\{E\} \quad (4)$$

where $\{T\} = \{T_{11}\ T_{22}\ T_{33}\ T_{12}\ T_{13}\ T_{23}\}^T$ is the stress vector, $\{S\} = \{S_{11}\ S_{22}\ S_{33}\ S_{12}\ S_{13}\ S_{23}\}^T$ the deformation vector, $\{E\} = \{E_1\ E_2\ E_3\}$ the electric field, $\{D\} = \{D_1\ D_2\ D_3\}$ the electric displacement, $[c]$ and $[s]$ the elasticity constants matrices, $[\varepsilon]$ the dielectric constants, $[d]$ and $[e]$ the piezoelectric constants. (superscripts ^{E}, ^{S} et ^{T} indicate values at E, S and T constant respectively)

2.2. Single Layer in Plane Stress

We consider a shell structure with embedded piezoelectric patches covered with electrodes. The piezoelectric patches are parallel to the mid-plane and orthotropic in their plane. The electric field and electric displacement are assumed uniform across the thickness and aligned on the normal to the mid-plane (direction 3). With the plane stress hypothesis, the constitutive equations can be reduced to

$$\begin{Bmatrix} T_{11} \\ T_{22} \\ T_{12} \end{Bmatrix} = \left[c^E\right] \begin{Bmatrix} S_{11} \\ S_{22} \\ 2S_{12} \end{Bmatrix} - \begin{Bmatrix} e_{31} \\ e_{32} \\ 0 \end{Bmatrix} E \quad (5)$$

$$D = \{e_{31}\ e_{31}\ 0\}\{S\} + \varepsilon^S E \quad (6)$$

where $\left[c^E\right]$ is the stiffness matrix of the piezoelectric material in its othotropy axes. In writing Equ.(5), it has been assumed that the piezoelectric principal axes are parallel to the structural othotropy axes. The analytical form of $\left[c^E\right]$ can be found in any textbook on composite materials.

2.3. Laminate

A laminate is formed from several layers bonded together to act as a single layer material (Figure 1); the bond between two layers is assumed to be perfect, so that the displacements remain continuous across the bond.

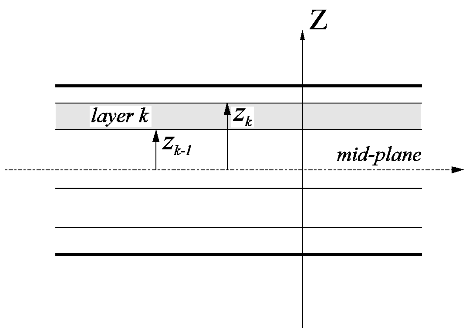

Figure 1: Multilayered material

2.3.1. Kirchhoff Plate

According to the *Kirchhoff* hypothesis, a fiber normal to the mid-plane remains so after deformation. It follows that:

$$\{S\} = \{S_0\} + z\{\kappa\} \quad (7)$$

where $\{S_0\}$ is the mid-plane deformation and $\{\kappa\}$, the mid-plane curvature. The constitutive equations for layer k in the global axes of coordinates read

$$\{T\} = \left[\overline{Q}\right]_k \{S\} - [R_T]_k^{-1} \begin{Bmatrix} e_{31} \\ e_{32} \\ 0 \end{Bmatrix}_k E_k \quad (8)$$

$$D_k = \{e_{31}\ e_{32}\ 0\}_k [R_S]_k \{S\} + \varepsilon_k E_k \quad (9)$$

where $[R_T]_k^{-1}$ is the transformation matrix relating the stresses in the local coordinate system (LT) to the global one (xy). Similarly, $[R_S]_k$ is the transformation matrix relating the strains in the global coordinate system (xy) to the local one (LT). The stiffness matrix of layer k in the global coordinate system, $\left[\overline{Q}\right]_k$, is related to $\left[c^E\right]_k$ by

$$\left[\overline{Q}\right]_k = [R_T]_k^{-1}\left[c^E\right]_k [R_S]_k \quad (10)$$

As mentioned before, the electric field E_k is assumed uniform across the thickness $h_k = z_k - z_{k-1}$ of the layer. Thus, we have $E_k = -\phi_k/h_k$, where ϕ_k is the difference of electric potential between the electrodes covering the surface on each side of the piezoelectric layer k.

The global constitutive equations of the laminate, which relate the resultant in-plane forces $\{N\}$ and bending moments $\{M\}$, to the mid-plane strain $\{S_0\}$ and curvature $\{\kappa\}$ and the potential applied to the various electrodes can now be derived by integrating Equ.(8) over the thickness of the laminate

$$\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} S_0 \\ \kappa \end{Bmatrix} + \sum_{k=1}^{n} \int_{z_{k-1}}^{z_k} \begin{bmatrix} I_3 \\ z\, I_3 \end{bmatrix} [R_T]_k^{-1} \begin{Bmatrix} e_{31} \\ e_{32} \\ 0 \end{Bmatrix}_k \frac{\phi_k}{h_k} dz \quad (11)$$

or

$$\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} S_0 \\ \kappa \end{Bmatrix} + \sum_{k=1}^{n} \begin{bmatrix} I_3 \\ z_{mk}\, I_3 \end{bmatrix} [R_T]_k^{-1} \begin{Bmatrix} e_{31} \\ e_{32} \\ 0 \end{Bmatrix}_k \phi_k \quad (12)$$

where

$$z_{mk} = \frac{z_{k-1} + z_k}{2} \quad (13)$$

is the distance from the mid-plane of layer k to the mid-plane of the laminate. The first term in the right hand side of Equ.(12) is the classical stiffness matrix of a composite laminate, where the extensional stiffness matrix $[A]$, the bending stiffness matrix $[D]$ and the extension/bending coupling matrix $[B]$ are related to the individual layers according to the classical relationships:

$$[A] = \sum_k \left[\overline{Q}\right]_k (z_k - z_{k-1})$$

$$[B] = \frac{1}{2}\sum_k \left[\overline{Q}\right]_k (z_k^2 - z_{k-1}^2)$$

$$[D] = \frac{1}{3}\sum_k \left[\overline{Q}\right]_k (z_k^3 - z_{k-1}^3) \quad (14)$$

The second term in the right hand side of Equ.(12) expresses the piezoelectric loading. Similarly, substituting Equ.(7) into Equ.(9), we get

$$D_k = \{e_{31}\ e_{32}\ 0\}_k [R_S]_k [I_3\ z\, I_3] \begin{Bmatrix} S_0 \\ \kappa \end{Bmatrix} - \varepsilon_k \frac{\phi_k}{h_k} \quad (15)$$

Since we have assumed that the electric displacement D_k is constant over the thickness of the piezoelectric layer, this equation can be averaged over the thickness, leading to

$$D_k = \{e_{31}\ e_{32}\ 0\}_k\, [R_S]_k\, [I_3\ z_{mk} I_3] \begin{Bmatrix} S_0 \\ \kappa \end{Bmatrix} - \frac{\varepsilon_k}{h_k}\phi_k \quad (16)$$

2.3.2. Mindlin Plate

The classical *Kirchhoff* theory neglects the transverse shear strains. Alternative theories which accomodate the transverse shear strains have been developed and have been found more accurate for thick shells (Hughes, 1987). In the *Mindlin* formulation, a fiber normal to the mid-plane remains straight, but no longer orthogonal to the mid-plane. Upon introducing the transverse shear strains $\{\gamma\} = \{\gamma_{xz}\ \gamma_{yz}\}^T$ and the transverse shear loads $\{T\} = \{T_{xz}\ T_{yz}\}^T$, the global constitutive equations of the piezoelectric *Mindlin* shell become

$$\begin{Bmatrix} N \\ M \\ T \end{Bmatrix} = \begin{bmatrix} A & B & 0 \\ B & D & 0 \\ 0 & 0 & K \end{bmatrix} \begin{Bmatrix} S_0 \\ \kappa \\ \gamma \end{Bmatrix} + \sum_{k=1}^{n} \begin{bmatrix} I_3 \\ z_{mk} I_3 \\ 0 \end{bmatrix} [R_T]_k^{-1} \begin{Bmatrix} e_{31} \\ e_{32} \\ 0 \end{Bmatrix}_k \phi_k \quad (17)$$

$$D_k = \{e_{31}\ e_{32}\ 0\}_k\, [R_S]_k\, [I_3\ z_{mk} I_3\ 0] \begin{Bmatrix} S_0 \\ \kappa \\ \gamma \end{Bmatrix} - \frac{\varepsilon_k}{h_k}\phi_k \quad (18)$$

where the load vector now includes the in-plane loads $\{N\}$ and moments $\{M\}$ and the transverse shear loads $\{T\}$. Similarly, the strain vector includes the mid-plane membrane strains $\{S_0\}$, the mid-plane curvatures $\{\kappa\}$ and the transverse shear strains $\{\gamma\}$. In Equ.(18), the stiffness matrices A, B & D are given by Equ.(14) while the transverse shear stiffness matrix K is obtained following the method described in (Batoz and Dhatt, 1990). We note that the transverse shear and the piezoelectric effect are entirely decoupled.

3. ACTUATION AND SENSING

3.1. Actuation: Equivalent Piezoelectric Loads

Equation (17) shows that a voltage ϕ applied between the electrodes of a piezoelectric patch produces in-plane loads and moments:

$$\begin{Bmatrix} N \\ M \end{Bmatrix} = -\begin{bmatrix} I_3 \\ z_m I_3 \end{bmatrix} [R_T]^{-1} \begin{Bmatrix} e_{31} \\ e_{32} \\ 0 \end{Bmatrix} \phi \quad (19)$$

If the piezoelectric properties are isotropic in the plane ($e_{31} = e_{32}$), we have

$$e_{31}\,[R_T]^{-1} \begin{Bmatrix} 1 \\ 1 \\ 0 \end{Bmatrix} = e_{31} \begin{Bmatrix} 1 \\ 1 \\ 0 \end{Bmatrix} \quad (20)$$

It follows that

$$\{N\} = \begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = -e_{31}\phi \begin{Bmatrix} 1 \\ 1 \\ 0 \end{Bmatrix} \quad (21)$$

$$\{M\} = \begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = -e_{31} z_m \phi \begin{Bmatrix} 1 \\ 1 \\ 0 \end{Bmatrix} \quad (22)$$

We note that the in-plane forces and the bending moments are both *hydrostatic*; they are independant of the orientation of the facet. We therefore conclude that the piezoelectric loads result in a uniform in-plane load N_p and bending moment M_p acting normally to the contour of the electrode as indicated Figure 2:

$$N_p = -e_{31}\phi,\ M_p = -e_{31} z_m \phi \quad (23)$$

where z_m is the distance from the mid-plane of the piezoelectric patch to the mid-plane of the plate.

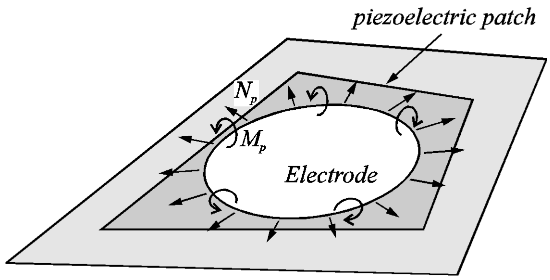

Figure 2: Piezoelectric load

3.2. Sensing

Consider a piezoelectric patch connected to a charge amplifier as in Figure 3. The charge amplifier imposes $\phi = 0$ between the electrodes and the output voltage is proportional to the electric charge:

$$\phi_{out} = -\frac{Q}{C_r} = -\frac{1}{C_r}\int_\Omega D\, d\Omega \quad (24)$$

where D is given by Equ.(18). If the piezoelectric properties are isotropic in the plane ($e_{31} = e_{32}$), we have

$$e_{31}\,\{1\ 1\ 0\}\,[R_S] = e_{31}\,\{1\ 1\ 0\} \quad (25)$$

and Equ.(24) becomes

$$\phi_{out} = -\frac{e_{31}}{C_r}\left[\int_\Omega \left(S_x^0 + S_y^0\right) d\Omega + z_m \int_\Omega (\kappa_x + \kappa_y)\, d\Omega\right] \quad (26)$$

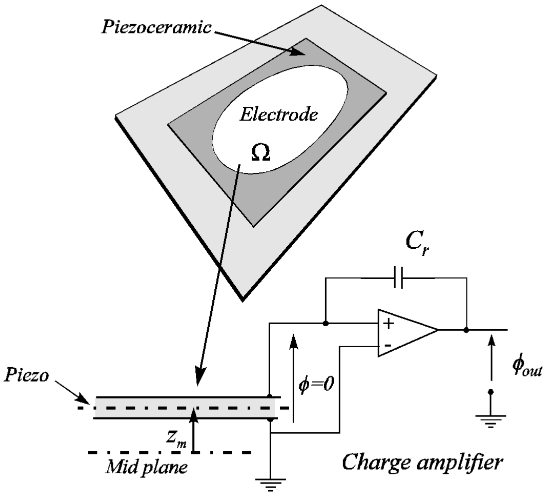

Figure 3: Piezoelectric sensor

The first integral represents the contribution of the average membrane strains over the electrode and the second, the contribution of the average bending moment. Using the *Green* integral

$$\int_{\Omega} \nabla.\mathbf{a}\, d\Omega = \int_{\mathcal{C}} \mathbf{a}.\mathbf{n}\, dl \qquad (27)$$

the foregoing result can be transformed into

$$\phi_{out} = -\frac{e_{31}}{C_r}\left[\int_{\mathcal{C}} \mathbf{u^0}.\mathbf{n}\, dl + z_m \int_{\mathcal{C}} \frac{\partial w}{\partial n}\, dl\right] \qquad (28)$$

where the integrals extend to the contour of the electrode. The first term is the mid-plane displacement normal to the contour while the second is the slope of the mid-plane in the plane normal to the contour (Figure 4). It is worth insisting that for both the actuator and the sensor, it is not the shape of the piezoelectric patch that matters, but rather the shape of the electrodes.

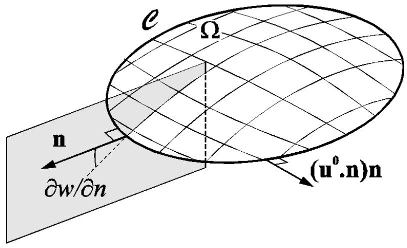

Figure 4: Contribution to the output of the piezoelectric isotropic sensor ($e_{31} = e_{32}$)

4. FINITE ELEMENT FORMULATION

The dynamic equations of a piezoelectric continuum can be derived from the *Hamilton* principle, in which the Lagrangian and the virtual work are properly adapted to include the electrical contributions as well as the mechanical ones. The potential energy density of a piezoelectric material includes contributions from the strain energy and from the electrostatic energy (Tiersten, 1967).

$$H = \frac{1}{2}\left[\{S\}^T\{T\} - \{E\}^T\{D\}\right] \qquad (29)$$

Similarly, the virtual work density reads

$$\delta\mathcal{W} = \{\delta u\}^T\{F\} - \delta\phi\, \sigma \qquad (30)$$

where $\{F\}$ is the external force and σ is the electric charge. From Equ.(29) and (30), the analogy between electrical and mechanical variables can be deduced (Table 1).

Mechanical		*Electrical*	
Force	$\{F\}$	σ	*Charge*
Displ.	$\{u\}$	ϕ	*Voltage*
Stress	$\{T\}$	$\{D\}$	*Electric Displ.*
Strain	$\{S\}$	$\{E\}$	*Electric Field*

Table 1: Electromechanical analogy

The variational principle governing the piezoelectric materials follows from the substitution of H and $\delta\mathcal{W}$ into the *Hamilton* principle (Allik and Hughes, 1970). For the specific case of the piezoelectric shell, we can write the potential energy

$$H = \frac{1}{2}\int_{\Omega}\left[\left\{S_0^T\ \kappa^T\ \gamma^T\right\}\begin{Bmatrix} N \\ M \\ T \end{Bmatrix} - E\ D\right] d\Omega \qquad (31)$$

Upon substituting Equ.(17) and (18) into Equ.(31), one gets the expression of the potential energy for a piezoelectric *Mindlin* shell. The element used is the *Mindlin* shell element from *Samcef* (*Samtech s.a.*). The electrical degrees of freedom are the voltages ϕ_k across the piezoelectric layers; it is assumed that the potential is constant over each element (this implies that the finite element mesh follows the shape of the electrodes). Introducing the matrix of the shape functions $[N]$ (relating the displacement field to the nodal displacements $\{q\}$), and the matrix $[\mathcal{B}]$ of their derivatives (relating the strain field to the nodal displacements), into the *Hamilton* principle and integrating by part with respect to time, we get

$$\begin{aligned} 0 = \quad & \{\delta q\}^T \int_V \rho[N]^T[N]dV\ \{\ddot{q}\} \\ + \ & \{\delta q\}^T \int_{\Omega} [\mathcal{B}]^T \begin{bmatrix} A & B & 0 \\ B & D & 0 \\ 0 & 0 & K \end{bmatrix} [\mathcal{B}]d\Omega\ \{q\} \end{aligned}$$

$$
\begin{aligned}
&+\ \{\delta q\}^T \int_\Omega [\mathcal{B}]^T \begin{bmatrix} ... & \mathcal{E}_k^T & ... \\ ... & \mathcal{E}_k^T z_{mk} & ... \\ ... & 0 & ... \end{bmatrix} d\Omega \begin{Bmatrix} \vdots \\ \phi_k \\ \vdots \end{Bmatrix} \\
&+\ \{...\ \delta\phi_k\ ...\} \int_\Omega \begin{bmatrix} \vdots & \vdots & \vdots \\ \mathcal{E}_k & \mathcal{E}_k z_{mk} & 0 \\ \vdots & \vdots & \vdots \end{bmatrix} [\mathcal{B}] d\Omega\ \{q\} \\
&+\ \{...\ \delta\phi_k\ ...\} \int_\Omega \begin{bmatrix} \ddots & & 0 \\ & \varepsilon_k/h_k & \\ 0 & & \ddots \end{bmatrix} d\Omega \begin{Bmatrix} \vdots \\ \phi_k \\ \vdots \end{Bmatrix} \\
&-\ \{\delta q\}^T \int_\Omega [N]^T \{P_S\} d\Omega - \{\delta q\}^T \{P_c\} \\
&+\ \{...\ \delta\phi_k\ ...\} \{...\ \sigma_k\ ...\}^T \qquad (32)
\end{aligned}
$$

where we have introduced

$$\{\mathcal{E}\}_k = \{e_{31}\ e_{32}\ 0\}_k\,[R_S]_k \qquad (33)$$

and we have used the fact that

$$[R_T]_k^{-1}\,\{e_{31}\ e_{32}\ 0\}_k^T = \{\mathcal{E}\}_k^T \qquad (34)$$

$[P_S]$ and $[P_c]$ are respectively the external distributed forces and concentrated forces and $\{...\ \delta\phi_k\ ...\}\,\{...\ \sigma_k\ ...\}^T = \sum_k \delta\phi_k\sigma_k$ is the electrical work done by the external charges σ_k brought to the electrodes.

Equation (32) must be verified for any $\{\delta q\}$ and $\{\delta\phi\}$ compatible with the boundary conditions; It follows that, for any element, we have

$$
\begin{aligned}
[M_{qq}]\{\ddot{q}\} + [K_{qq}]\{q\} + [K_{q\phi}]\{\phi\} &= \{f\} \qquad (35)\\
[K_{\phi q}]\,\{q\} + [K_{\phi\phi}]\{\phi\} &= \{g\} \qquad (36)
\end{aligned}
$$

where the element mass, stiffness, coupling and piezoelectric capacitance matrices are defined as

$$
\begin{aligned}
[M_{qq}] &= \int_V \rho [N]^T [N] dV \qquad (37)\\
& \qquad (38)\\
[K_{qq}] &= \int_\Omega [\mathcal{B}]^T \begin{bmatrix} A & B & 0 \\ B & D & 0 \\ 0 & 0 & K \end{bmatrix} [\mathcal{B}] d\Omega \qquad (39)\\
[K_{q\phi}] &= \int_\Omega [\mathcal{B}]^T \begin{bmatrix} ... & \mathcal{E}_k^T & ... \\ ... & \mathcal{E}_k^T z_{mk} & ... \\ ... & 0 & ... \end{bmatrix} d\Omega \qquad (40)\\
[K_{\phi\phi}] &= \Omega \begin{bmatrix} \ddots & & 0 \\ & \varepsilon_k/h_k & \\ 0 & & \ddots \end{bmatrix} \qquad (41)\\
[K_{\phi q}] &= [K_{q\phi}]^T \qquad (42)
\end{aligned}
$$

and the external mechanical forces and electric charge:

$$
\begin{aligned}
\{f\} &= \int_\Omega [N]^T \{P_S\} d\Omega + \{P_c\} \\
\{g\} &= -\{...\ \sigma_k\ ...\}^T
\end{aligned}
$$

The element coordinates $\{q\}$ and $\{\phi\}$ are related to the global coordinates $\{Q\}$ and $\{\Phi\}$. The assembly takes into account the equipotentiality condition of the electrodes; this reduces the number of electric variables to the number of electrodes. Upon carrying out the assembly, we get the global system of equations

$$
\begin{aligned}
[M_{QQ}]\{\ddot{Q}\} + [K_{QQ}]\{Q\} + [K_{Q\Phi}]\{\Phi\} &= \{F\} \qquad (43)\\
[K_{\Phi Q}]\,\{Q\} + [K_{\Phi\Phi}]\{\Phi\} &= \{G\} \qquad (44)
\end{aligned}
$$

where the global matrices can be derived in a straightforward manner from the element matrices (37) to (42). As for the element matrices, the global coupling matrices satisfy $[K_{\Phi Q}]=[K_{Q\Phi}]^T$.

5. ELECTRICAL BOUNDARY CONDITIONS

Equations (43) and (44) couple the mechanical variables $\{Q\}$ and the electrical potentials $\{\Phi\}$ between the electrodes of the piezoelectric patches; $\{F\}$ represents the external forces applied to the structure and $\{G\}$ the electric charges brought to the electrodes.

5.1. Voltage Driven Electrodes

If the electric potential $\{\Phi\}$ is controlled, the governing equations become

$$[M_{QQ}]\{\ddot{Q}\} + [K_{QQ}]\{Q\} = \{F\} - [K_{Q\Phi}]\,\{\Phi\} \qquad (45)$$

where the second term in the right hand side represents the equivalent piezoelectric loads. Once the mechanical displacements have been computed, the electric charges appearing on the electrodes can be computed from Equ.(44). From Equ.(45), we see that the eigenvalues problem of the system with short-circuited electrodes ($\{\Phi\} = 0$) is:

$$\left([K_{QQ}] - \omega^2 [M_{QQ}]\right)\{Q\} = 0 \qquad (46)$$

5.2. Charge Driven Electrodes

Conversely, open electrodes correspond to a charge condition $\{G\} = 0$. In this case, Equ.(44) becomes

$$\{\Phi\} = -[K_{\Phi\Phi}]^{-1}[K_{\Phi Q}]\{Q\} \qquad (47)$$

and, upon substituting into Equ.(43), we get

$$[M_{QQ}]\{\ddot{Q}\} + [K]^*\{Q\} = \{F\} \tag{48}$$

with

$$[K]^* = [K_{QQ}] - [K_{Q\Phi}][K_{\Phi\Phi}]^{-1}[K_{\Phi Q}] \tag{49}$$

which shows that the stiffness matrix depends on the electrical boundary conditions.

5.3. Electrodes Connected via a Passive Network

If the electrodes are connected via a passive electrical network of impedance matrix $[\mathcal{Z}]$, Equ.(43) and (44) must be complemented by the network equation (Figure 5). Since the electrical current is given by the time derivative of the electrical charge $\{I\} = \{\dot{G}\}$, we have, in *Laplace* notations:

$$\{\Phi\} = -[\mathcal{Z}]\{I\} = -[\mathcal{Z}(s)]s\{G\} \tag{50}$$

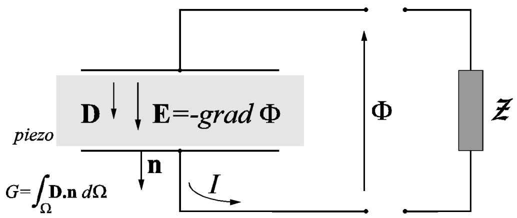

Figure 5: Electrodes connected to an impedance

6. CONCLUSION

The theory of piezolaminated plates has been developed; the fundamental equations governing the equivalent piezoelectric loads of a piezoelectric actuator and the output of a piezoelectric sensor have been derived. Numerous validation benchmarks have been carried out. Some are described in (Piéfort and Preumont, 1998).

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